Electrical Metrology

DC & AC Power/Energy Measurements

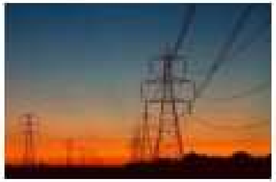
MEASUREMENT
& STANDARDS
Keys to COMPETITIVENESS
and A SAFER WORLD

Laboratoire national de métrologie et d’essais

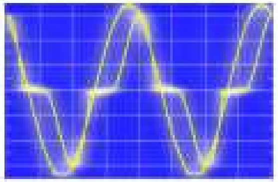
Context

Being transported, distributed then sold to domestic users or industrials, the electricity “product” invoices the number of kilowatt-hours consumed ⇒ measurement traceability owing to high quality **power and energy standards** + **electrical power and energy quality EPEQ** (= quality of the voltage supplied by the grid).

Renewable Energy Performance



Transmission & Distribution Efficiency



Power Quality Compliance

Energy Saving Evaluation

Accurate Electricity Metering

Main disturbances

- voltage dips,
- harmonics and interharmonics,
- voltage fluctuations or flicker,
- swells, temporary or transient overvoltages...

Electric power measurements




Multiplication of instruments dedicated to **electricity analysis**: qualimeter, disturbancegraph, oscilloperturbogragh quality analyzer, grid analyzer, power analyzer….




⇒ Development of **new primary standards** in NMIs

Use of sampling technics -> Uncertainties : 10 ppm

- Instantaneous power, p(t)

Expression of power p(t):

$$p(t) = u(t).i(t)$$

instantaneous power (watt)

where u(t) is the voltage and i(t) the current

- Mean power, P

The mean power, over a time interval T, is :

$$P = 1/T \int_0^T u(t).i(t).dt$$

if $v(t) = V \cos \omega t$ and $i(t) = I \cos (\omega t - \varphi)$ then ,

$p(t) = VI \cos \omega t \cos (\omega t - \varphi)$ and $P = VI/2. \cos \varphi = V_{In} I_{In}. \cos \varphi$,

where V_{In} et I_{In} are r.m.s. values

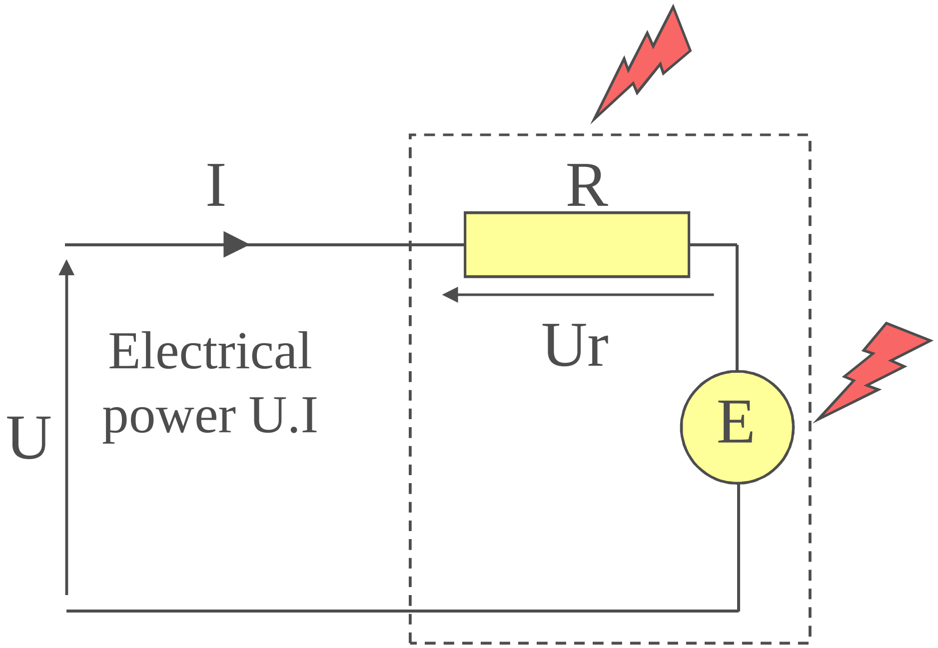
Electrical Power in DC current

The load seen by the source is purely resistive

Heat dissipated

$P = U_r.I = R.I^2 \Leftrightarrow$ Heat

In DC current, the power is the product of the voltage at the terminals of the resistive load multiplied by the current passing through it.



Mechanical power

$P = E.I \Leftrightarrow$ couple, angular speed

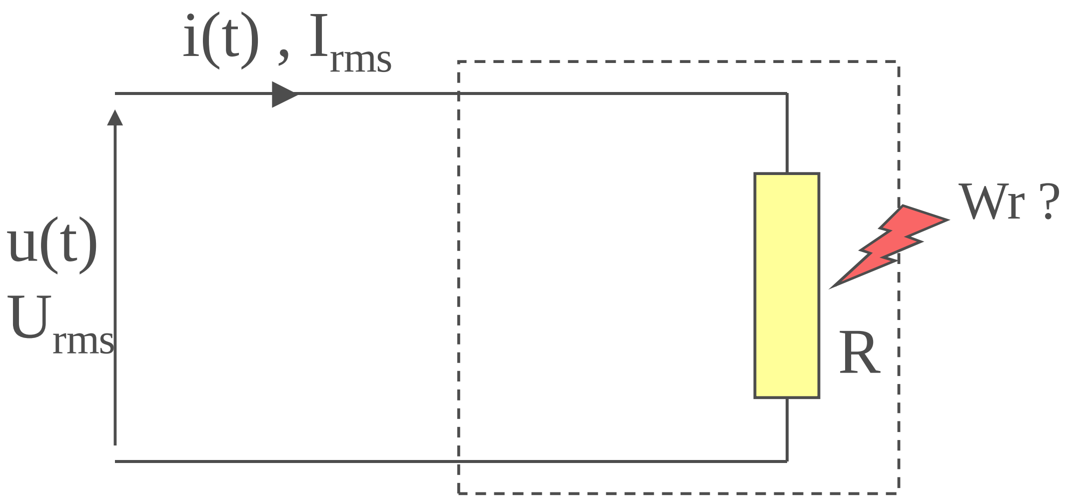
Rotor of an electric motor for DC current

In DC current, $p(t) = P = cst$

Unit : watt

Electrical Power in AC current

Particular case : a pure resistive load



$$Wr = \frac{1}{T}.\int_0^T \overbrace{u(t).i(t)}^{\text{Instantaneous power dissipated in R}}.dt$$

$$Wr = \frac{R}{T}.\int_0^T i^2(t).dt$$

$$Wr = R.I_{rms}^2 = U_{rms}.I_{rms}$$

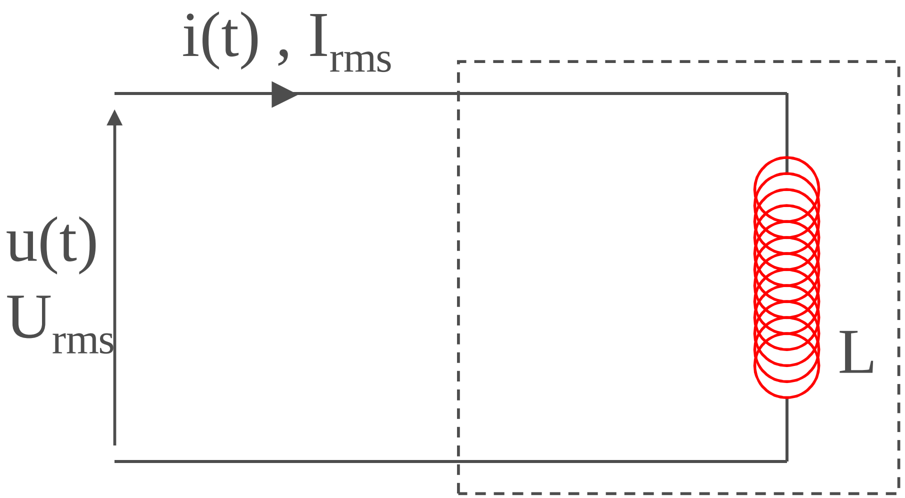
The electric power provided to the resistance is completely dissipated into heating (as in DC current).

$$P = U_{rms}.I_{rms}$$

P is also the « active power » (unit : watt) (reactive power = 0)

Electrical Power in AC current

Particular case : a pure inductive load (1/2)

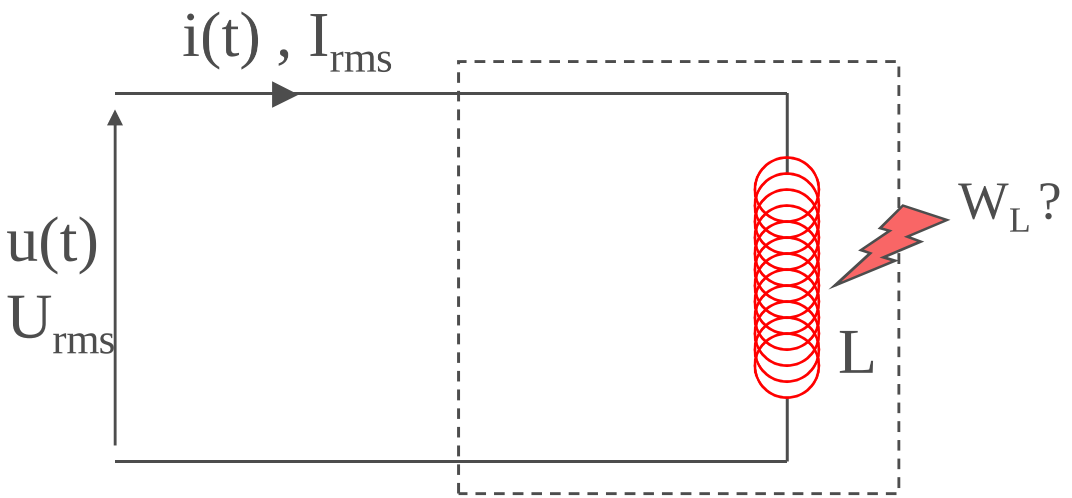


By analogy, the electric power apparently provided to the inductance is equal to the $U_{rms}.I_{rms}$ product.

This quantity is called apparent power "S" (unit: SA).

Electrical Power in AC current

Particular case : a pure inductive load (2/2)



$$U_L(t) = L.\frac{di(t)}{dt}$$

$$W_L = \frac{1}{T}\int_0^T L.\frac{di(t)}{dt}.i(t)dt$$

$$W_L = \frac{L}{T}\left[\frac{i^2(t)}{2}\right]_0^T = \frac{L}{T}.\left(\frac{i^2(T)-i^2(0)}{2}\right)$$

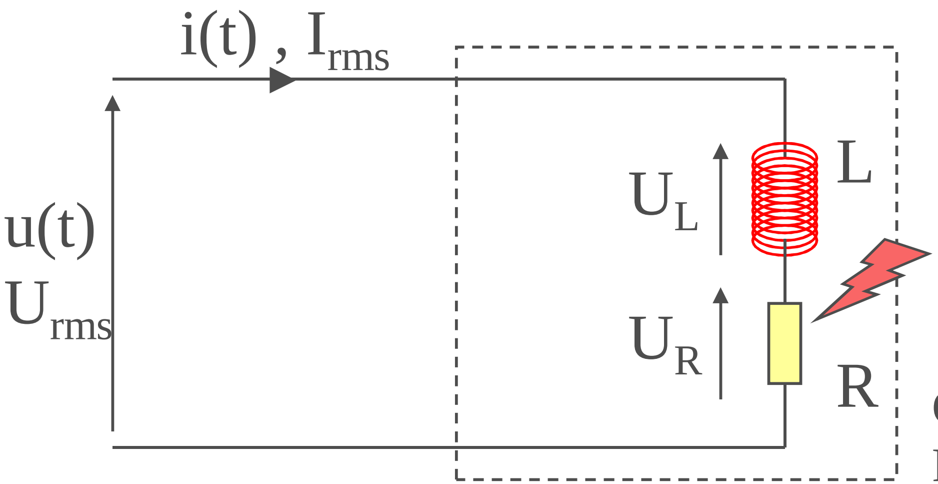
$$W_L = 0$$

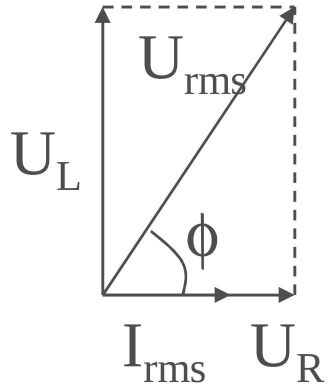
Over one period, in a purely inductive circuit, the mean power (active power) is equal to 0 (the same behavior for a capacitive circuit).
The electric energy provided by the source is stored by the inductor when the current increases then given back when the current decreases.

The apparent power ("apparently consumed by inductance") is also :
the « reactive power Q » (unit : VAR) (active power = 0)

Electrical Power in AC current

Unspecified load





$Q = U_L.I_{rms} = U_{rms}.I_{rms}.\sin\phi$ ⇨ Reactive power

$P = U_R.I_{rms} = U_{rms}.I_{rms}.\cos\phi$ ⇨ Active power

For an unspecified load, the provided apparent power have two components : the active power (exploitable) and the reactive power (not exploitable).

- Active power $P = S.\cos\phi$ (real part)
- Reactive power $Q = S.\sin\phi$ (imaginear part)

with S, the provided apparent power

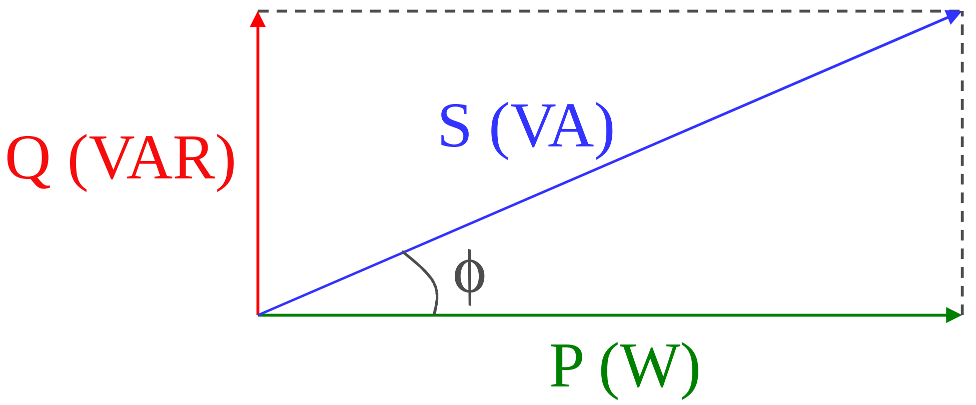
Vectorial representation

For an unspecified load, the provided apparent power have two components : the active power (exploitable) and the reactive power (not exploitable).

- Active power $P = S.\cos\phi$ (real part)
- Reactive power $Q = S.\sin\phi$ (imaginary part)

with S, the provided apparent power

Vectorial representation



Electrical Power in AC current

Power factor

The power factor is the expression of the part of the active power (exploitable) contained in the apparent power (apparently provided by the source).

$$\text{Power factor} = \cos\phi = \frac{P}{S}$$

Generally : $0{,}8 < \cos\phi < 0{,}9$

$\cos\phi \searrow$
- the consumer is penalized
- energy losses due to Joule effects in HV lines

$\cos\phi \nearrow$
- interesting for the consumer and the distributor of energy

To improve the $\cos\phi$, capacitors are added in HV circuits : they compensate more particullary the selfic effects due to windings of the engines.

Power in AC sine regime

Summary

DC regime: Power in AC regime $P = U.I$ (Watt)

AC regime: Active power $P = U_{rms}.I_{rms}.\cos\phi$ (Watt)

Reactive power $Q = U_{rms}.I_{rms}.\sin\phi$ (VAR)

Apparent power $S = U_{rms}.I_{rms}$ (VA)

Power factor $F = \cos\phi$

From a metrological point of view …

Active power (it is the wanted power) must be calibrated.

In some cases, reactive power (it is the compensation of the circuit) must be measured.

Electrical power in distorted regime

- Fourier transform for periodic signals

$$v(t) = V_0 + A_1.\sin(\omega.t) + A_2.\sin(2.\omega.t) + \dots + A_k.\sin(k.\omega.t) + \dots$$
$$+ B_1.\cos(\omega.t) + B_2.\cos(2.\omega.t) + \dots + B_k.\cos(k.\omega.t) + \dots$$

$$V_0 = \frac{1}{k.T}\int_0^{k.T} v(t)\,dt$$

$$A_k = \frac{2}{k.T}\int_0^{k.T} v(t).\sin(k.\omega.t)\,dt$$

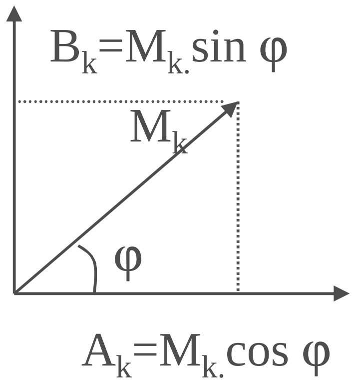
$$B_k = \frac{2}{k.T}\int_0^{k.T} v(t).\cos(k.\omega.t)\,dt$$

$$M_k.\sin(k.\omega. - \varphi_k)$$

with

$$M_k = \sqrt{A_k^{\;2} + B_k^{\;2}}$$

$$\varphi_k = \mathrm{atan}\left(\frac{B_k}{A_k}\right)$$



Electrical power in distorted regime

- Active power

$$P = \frac{1}{T}\int_0^T u(t).i(t)dt = P_0 + P_1 + P_2 + \ldots + P_k + \ldots$$

unit : Watt (W)

with :

$P_o=V_o.I_o$, power linked to an eventual continuous component
and
$P_k=V_k.I_k..\cos\phi_k$, mean power corresponding to harmonic of rank k

Particular Case : $u(t)=U.\sin\omega t$

$P = P_1$

Electrical power in distorted regime

- Reactive power (one definition)

$$\boxed{Q = Q_1 + Q_2 + \ldots + Q_k + \ldots}$$

unit : (VAR)

with :

$Q_k = V_k . I_k . \sin \phi_k$,mean power corresponding to harmonik of rank k

Particular case : $u(t) = U . \sin \omega t$

$Q = Q_1$

Electrical power in distorted regime

- Reactive power (one definition)

Apparent power is equal to the product of rms values of both current and voltage

$$\boxed{S = \sqrt{\sum_{n} V_k^{\,2}} \cdot \sqrt{\sum_{n} I_k^{\,2}} > \sqrt{P^2 + Q^2}}$$ (Cf. Example)

In order to keep a relationship between power components, a new type of power component is then introduced :

Deforming power $$D = \sqrt{S^2 - P^2 - Q^2}$$

As a consequence $$\boxed{S = \sqrt{P^2 + Q^2 + D^2}}$$

Non sinusoidal voltage and current periodic signals

$$\boxed{P = P_0 + P_1 + P_2 + \ldots + P_k + \ldots}$$ unit: watt (W)

with :

$P_o = V_o.I_o$, direct component, where applicable

and

$P_k = V_k.I_k.\cos \varphi_k$, mean power corresponding to each harmonic number

In non-sinusoidal conditions, the power factor does not depend directly and exclusively on the phase difference between voltage and current.

Power in three-phase network (in a balanced circuit)

$$P = 3.V_{in}.I_{in}.\cos\varphi = \sqrt{3}.U_{in}.I_{in}.\cos\varphi$$

$$Q = 3.V_{in}.I_{in}.\sin\varphi = \sqrt{3}.U_{in}.I_{in}.\sin\varphi$$

$$S = 3.V_{in}.I_{in} = \sqrt{3}.U_{in}.I_{in}$$

where P, Q, S are active, reactive and apparent power

V_{ln} is the r.m.s. value of the phase-to-phase voltage between conductors and neutral (simple voltage)
U_{ln} is the r.m.s. value of the phase voltage between 2 conductors (compound voltage)
I_{ln} is the r.m.s. value of the current in a conductor

Techniques for measuring an AC power

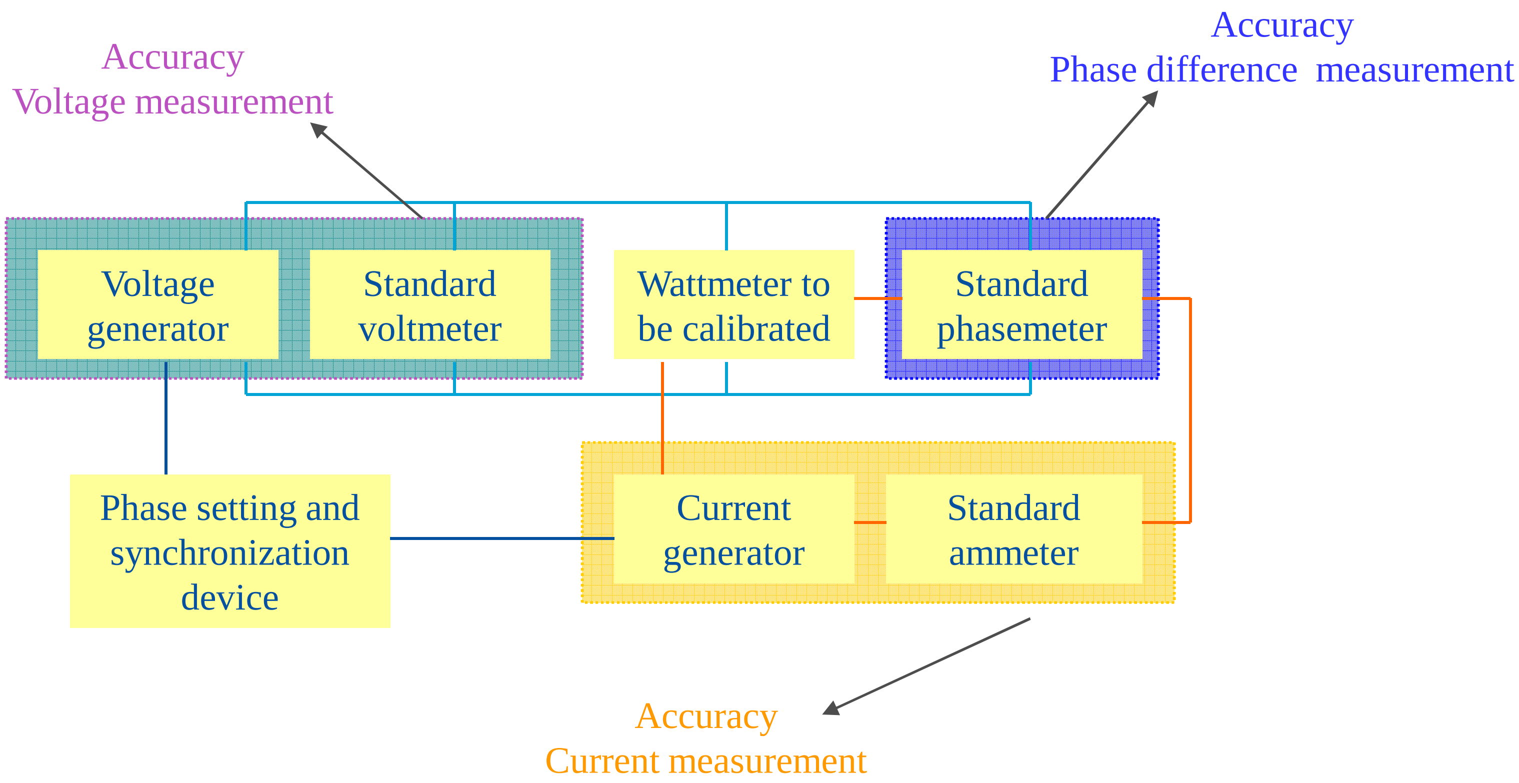
Separate measurements

Separate measurements of U_{rms}, I_{rms} and ϕ

$$S = UI$$

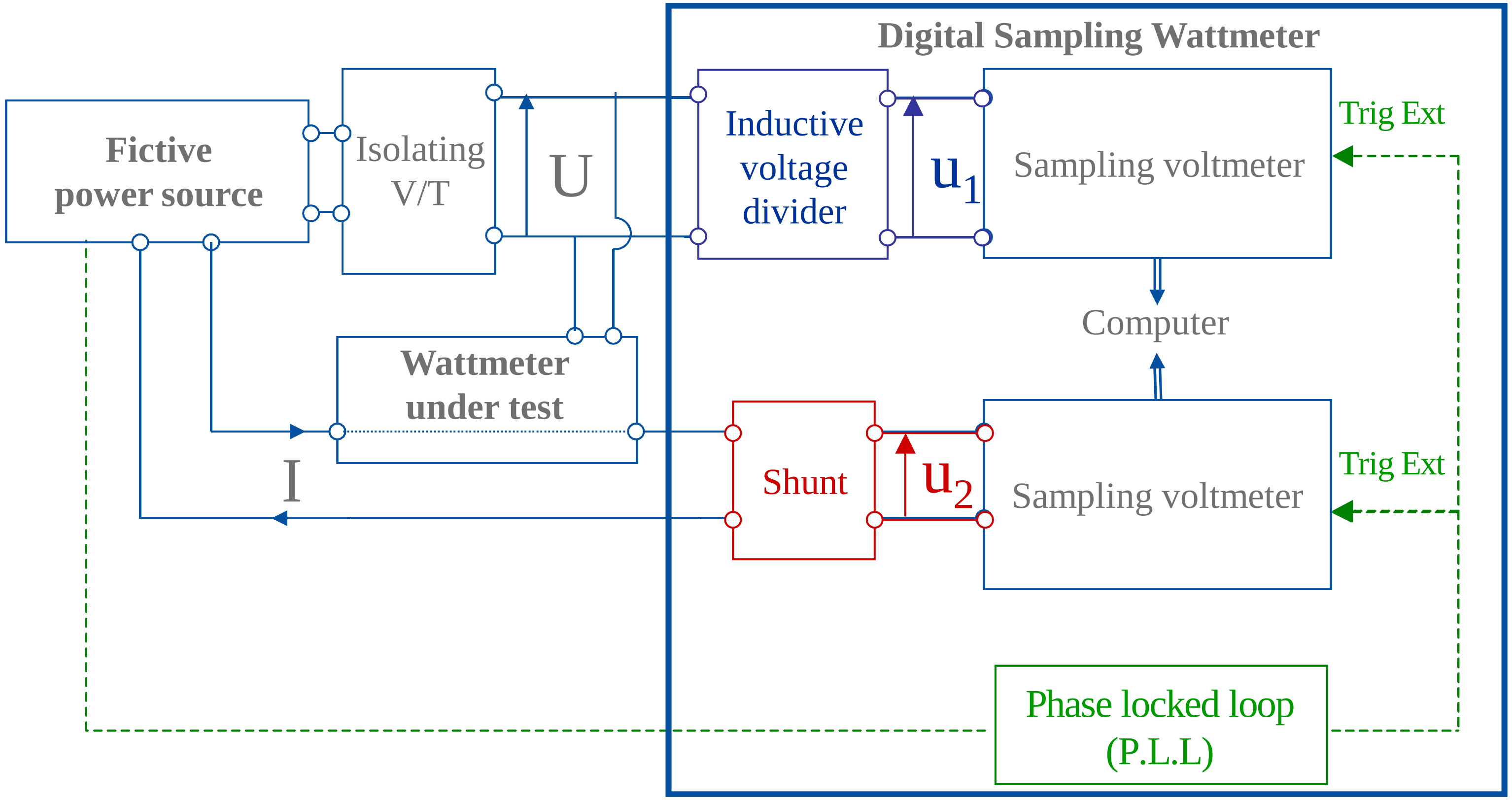
$$P = UI \cos\phi$$

$$Q = UI \sin\phi$$



Separate measurements

Sampling methods (1/2)



Separate measurements

Sampling methods (2/2)

Uncertainty budget achieved at LNE (1σ) from 8 to 13 μW/VA
f =53 Hz, U = 120 V, I = 5 A, cos ϕ = 1, 0.5 i/c, 0.001 i/c

Different approaches

International comparison on power measurements in a single phase configuration

INTI (Argentina)	10 to 19	NSL (New Zealand)	19 to 22
NMIA (Australia)	6 to 8	VNIIM (Russia)	9 to 14
INMETRO (Bra)	25 to 30	NMC (Singapour)	20 to 40
NRC (Canada)	6 to 8	CENAM (Mexico)	17 to 27
NIM (China)	4 to 6	SP (Sweden)	8 to 15
PTB (Germany)	2,5 to 8	NPL (Great Britain)	13 to 16
IEN (Italy)	15	NIST (USA)	6 to 9
LNE (France)	10	KIM LIPI (Indonesia)	-

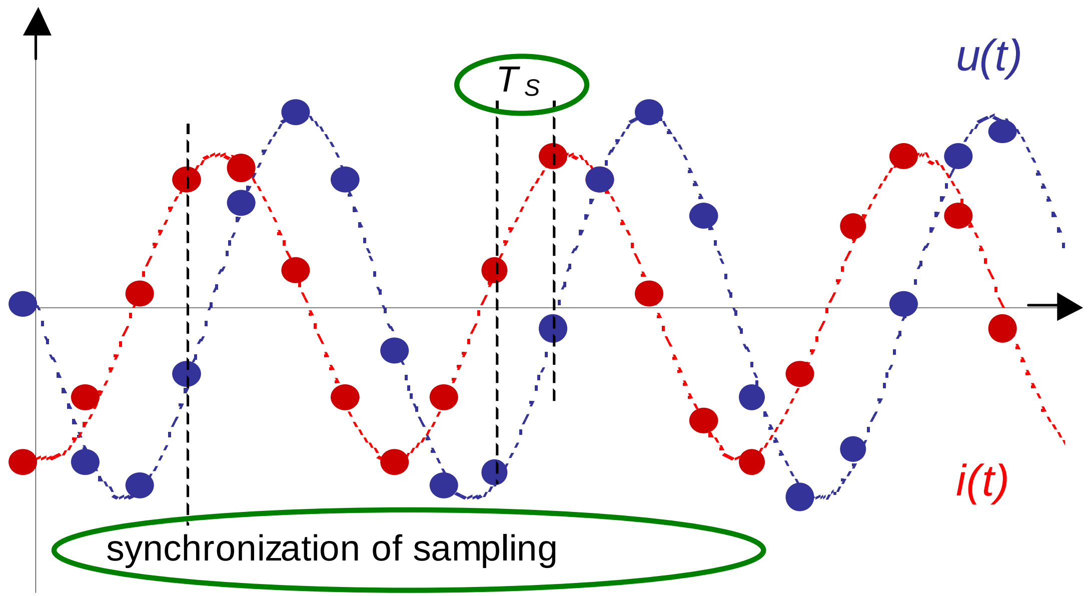
(3 methods)

Electrothermal comparison brigde

Sampling digital wattmeter

Current comparator

Digital sampling wattmeter



Sampling of signals
(sampling + quantization)
$\Rightarrow$ Analog to digital converters (ADC)

$\Downarrow$

Continuous signal $\rightarrow$ discrete signal
[sampling de *u(t)* et *i(t)*]
+
Discrete Fourier Transform of samples

$\Downarrow$

Amplitude spectra: U, I + phase spectra: ϕ

$\Downarrow$

$$\boxed{P = UI\cos\phi}$$

Digital sampling wattmeter

Although the **analog signal processing** of electrical signals continues to occupy an important place in metrology, it is more and more replaced by **digital signal processing** owing to:

- Mathematics (FFT algorithms),
- Electronics and Micro-electronics (ADC more and more efficient),
- Computer science (increased computing capabilities of microprocessors)

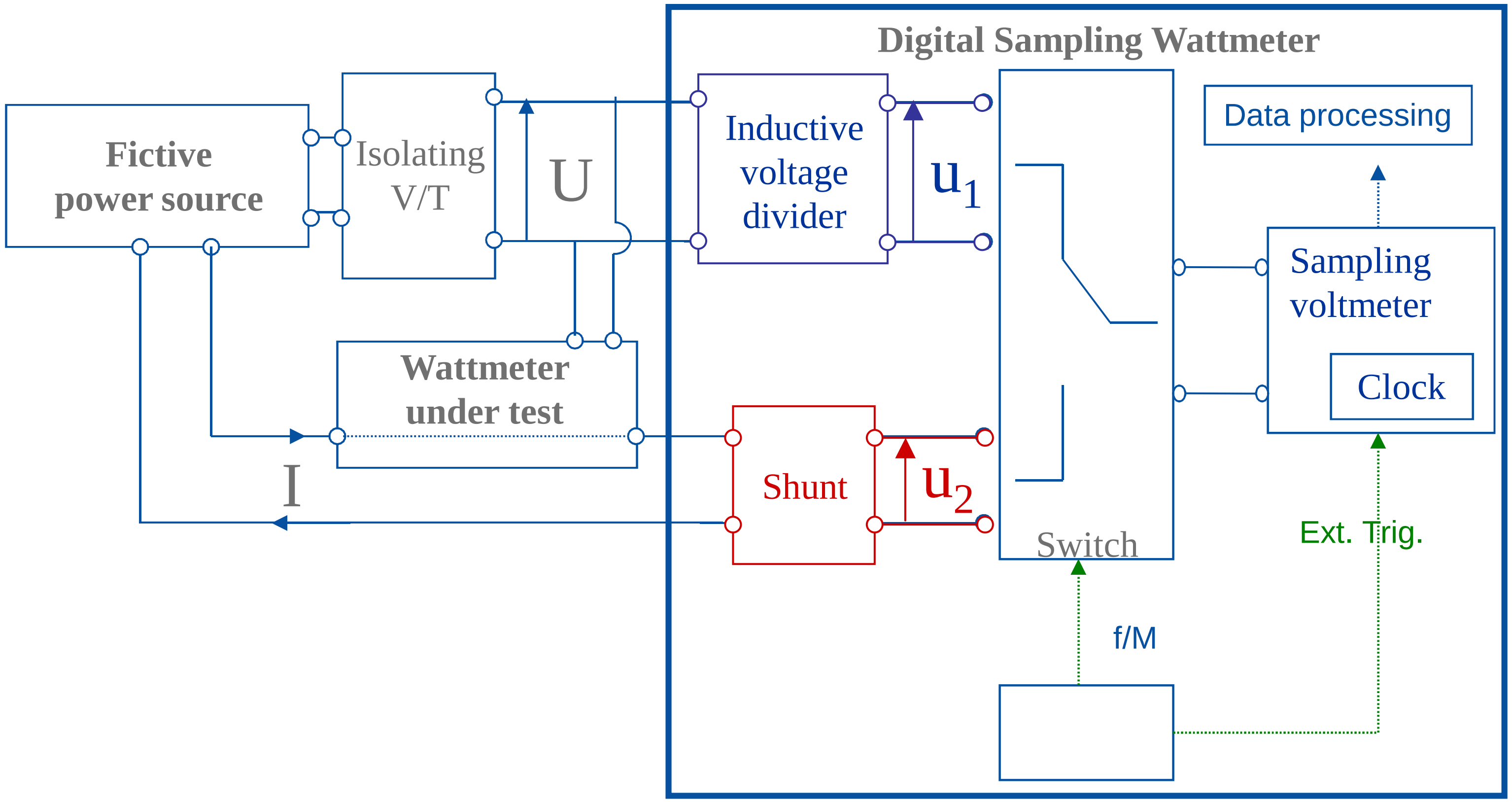
⇓

Example: **sampling digital wattmeter** developed instead of traditional methods.
These "new" wattmeters offer many advantages : **measurement acuracy**, **ease of implementation** and **low cost**. They also provide access to all the **features of both voltage and current** signals (amplitudes and phase angles of the fundamental component and those of harmonics) and thus allow the measurement of active power and reactive power **in sinusoidal regime** but also deformed regime **(**presence of harmonics**)**.

LNE
Sharing a passion for progress

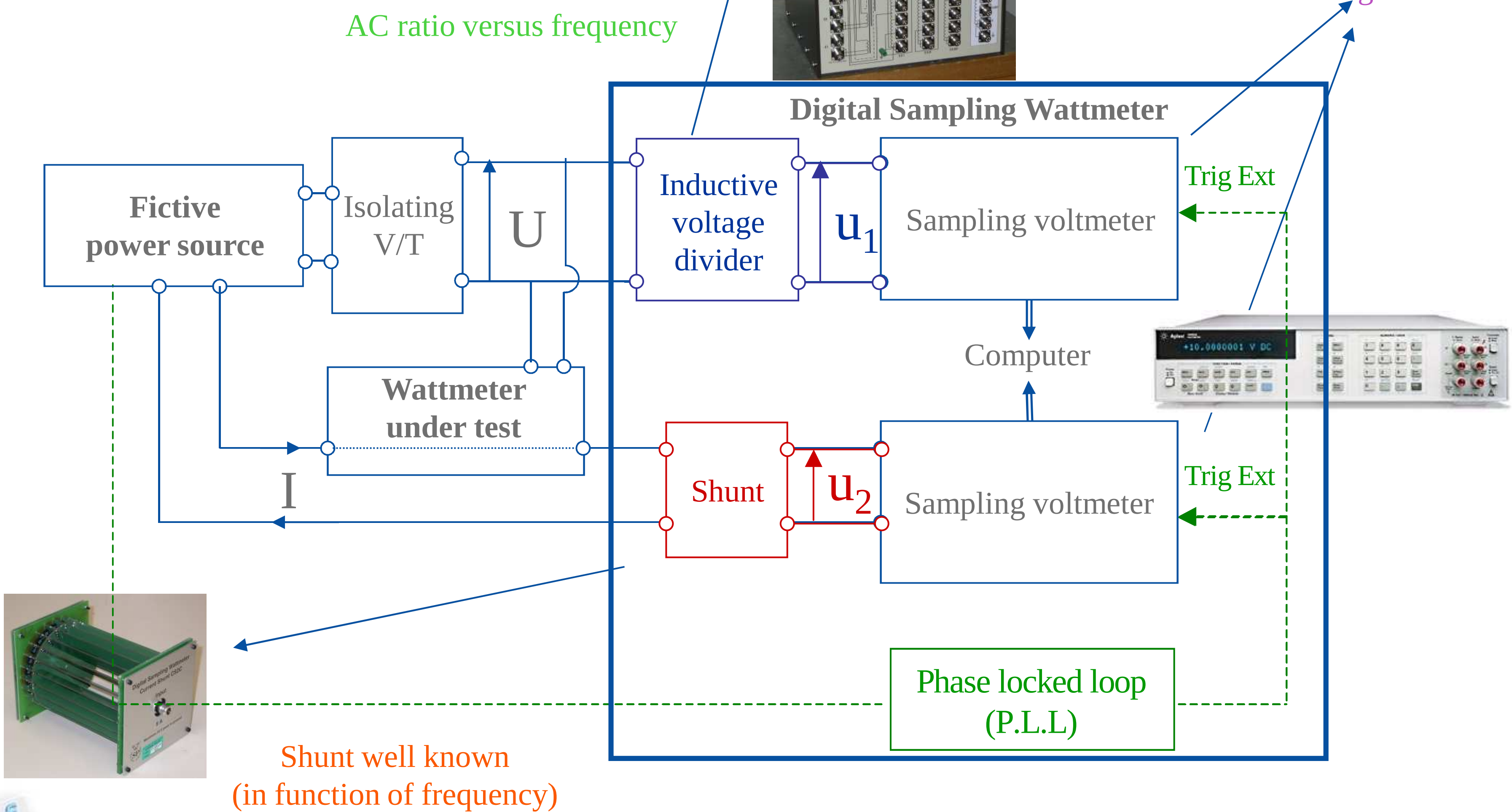
Digital sampling devices (1/18)

Uncertainties : 2.5 to 8 ppm



Digital sampling devices (1/18)

<u>Uncertainties : 10 ppm</u>



Digital sampling devices (2/18)

Shunts: partnership with SP (Sweden)

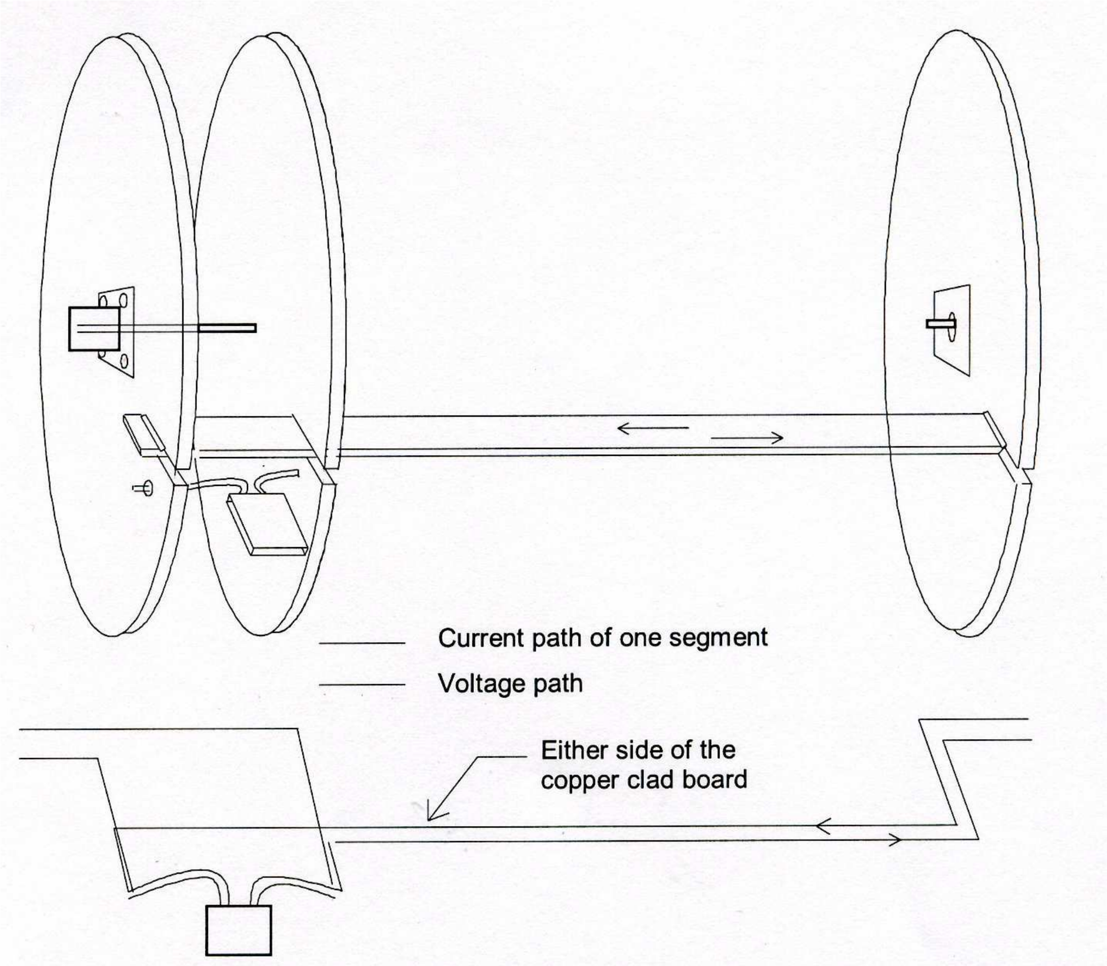
"Squirrel cage" structure
$\Rightarrow$ minimise **inductive coupling**

Specifications for 5 A / 0,8 V shunt
(45 Hz < f_0 < 65 Hz)

DC resistance
R = 160,0158.(1±1×10^{-6}) mΩ
AC/DC transfer difference
δ = (0 ± 10) µA/A
Phase shift
Δϕ = (0,4 ± 0,6) µrad

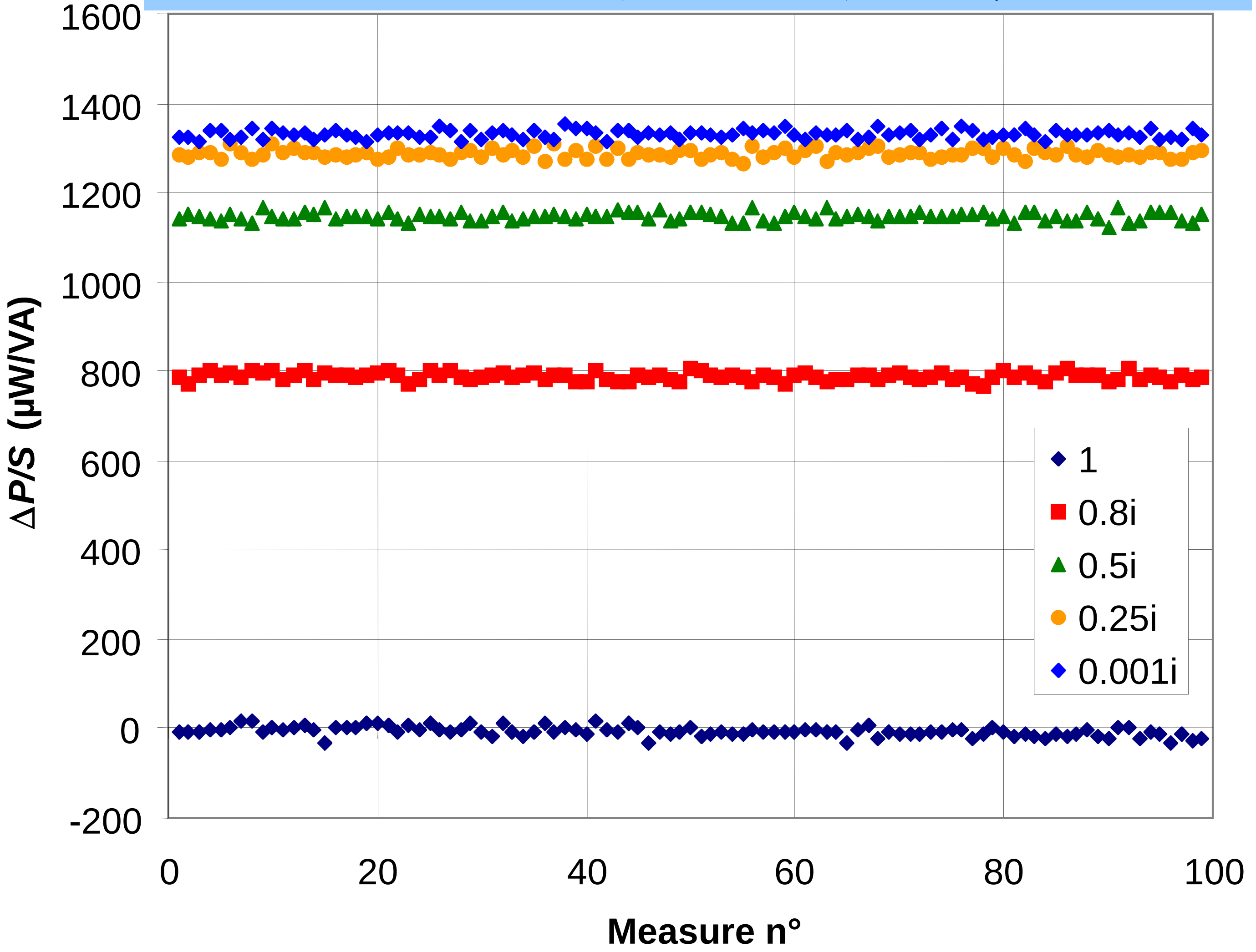
Digital sampling devices (2/18)

Shunts: partnership with SP (Sweden)



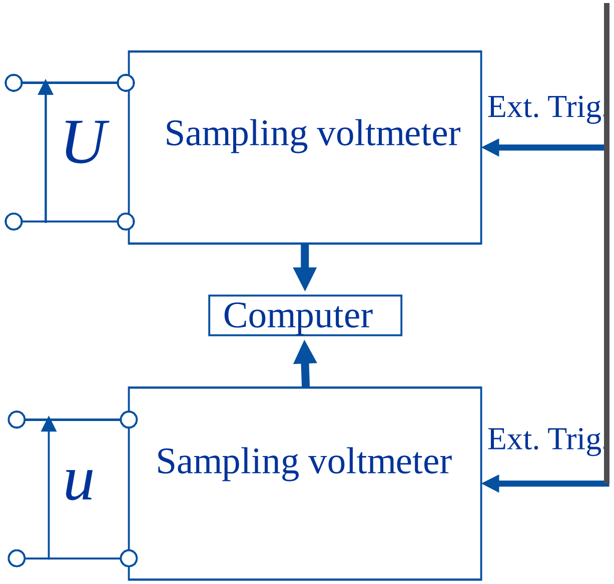
Digital sampling devices (3/18)

U = 120 V, I = 5 A (u = 0,8 V), cos ϕ variable



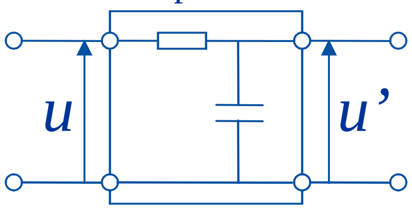
0°
36°
60°
75.5°
89.9°

Digital sampling devices (3/18)



120 V [1000 V] → $f_u = 30$ kHz

Low-pass filter



0,8 V [1 V] → $f_i = 150$ kHz

⇓

$$\left(\frac{\Delta P}{S}\right) = \sin\phi.\left(\frac{f}{f_u} - \frac{f}{f_i}\right)$$

(quadrature error)

⇐

cos ϕ	**$(\Delta P/S)_{theoretical}$**	**$(\Delta P/S)_{exp}$**
1	**0**	**-6**
0.8	**800**	**789**
0.5	**1154**	**1146**
0.25	**1290**	**1287**
0.001	**1333**	**1331**

⇓

Voltage divider

Digital sampling devices (4/18)

Voltage inductive divider: partnership with NMIA (Australia)

Specifications

Type : inductive voltage divider
Maximum input voltage : 1100 V ac rms
Transformer ratio $k = V_s/V_e$: 100 à 1000
Frequencies domain : 40 Hz à 1 kHz
Masse : 117 kg

Uncertainties (for V_{in} = 120 V and k = 200):

$$k \pm \sigma_k = 199{,}9999716 \pm 0{,}0000023$$

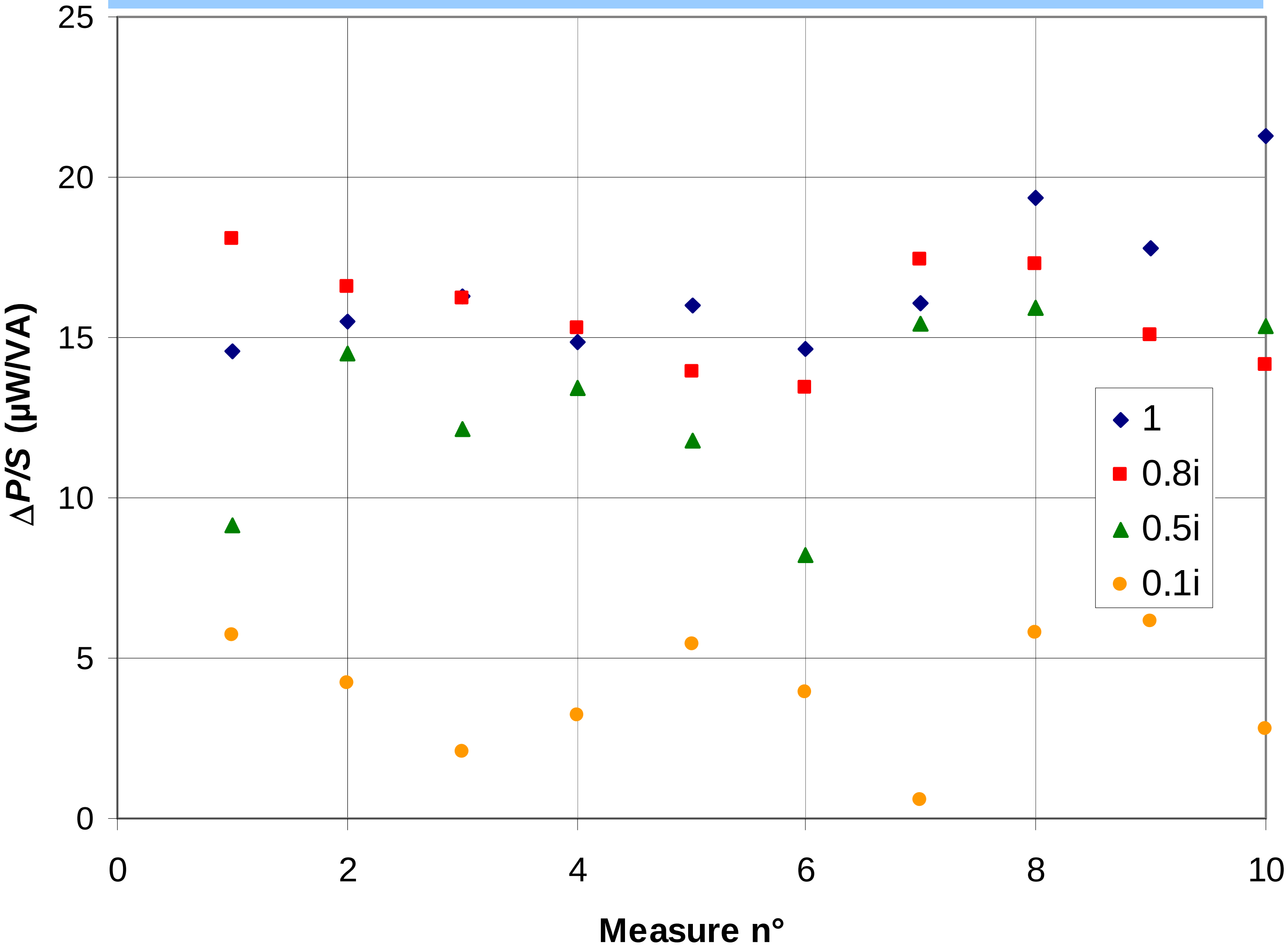
$$\frac{\sigma_k}{k} = \pm 1{,}2.10^{-8}$$

$$\phi_{IVD} \pm \sigma_{\phi_{IVD}} = (1{,}36 \pm 0{,}05)\,\mu rad$$

Digital sampling devices (4/18)

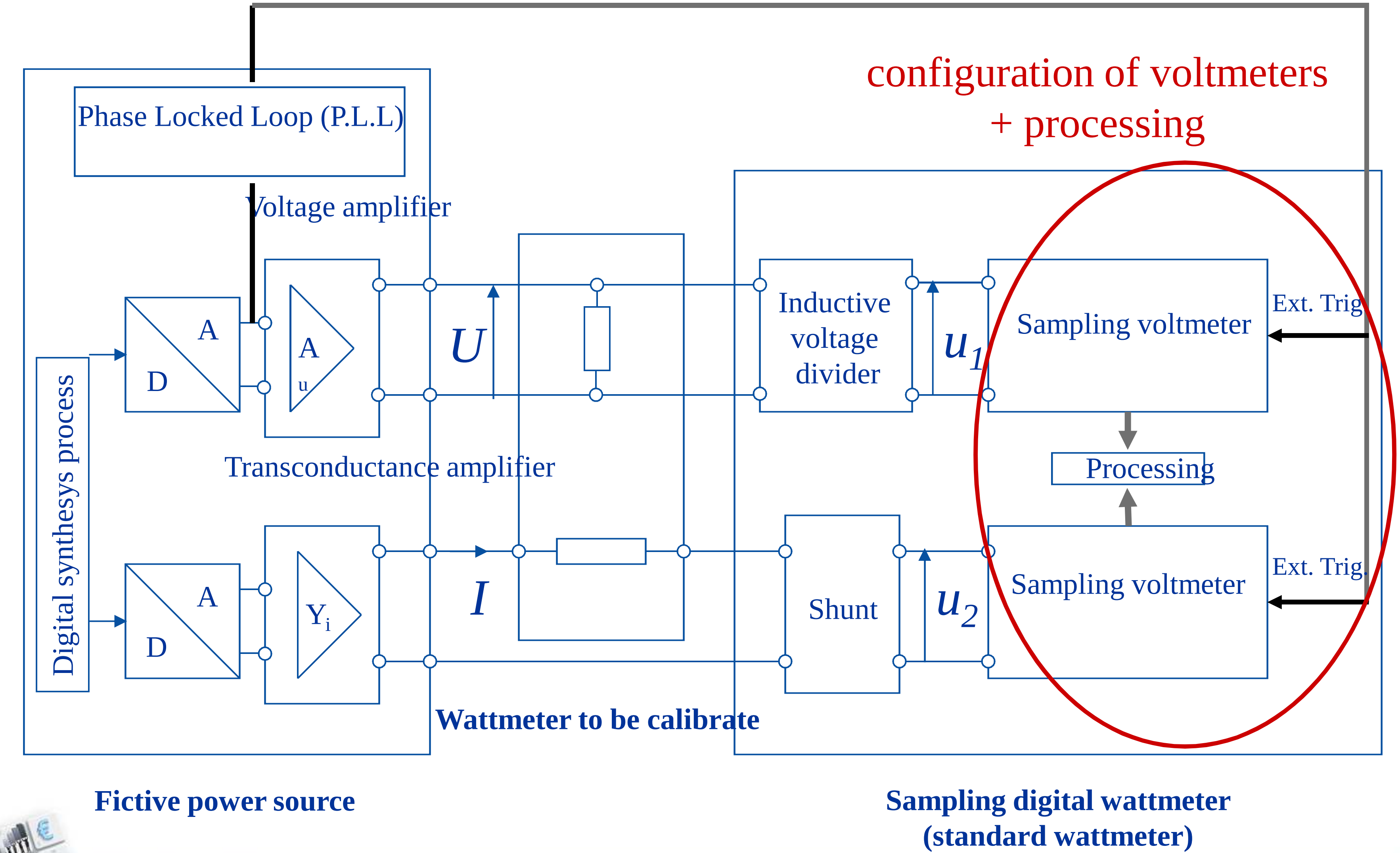
Voltage inductive divider: partnership with NMIA (Australia)

U = 120 V, I = 5 A (u = 0,8 V), cos ϕ variable



0°
36°
60°
84.3°

Digital sampling devices (5/18)

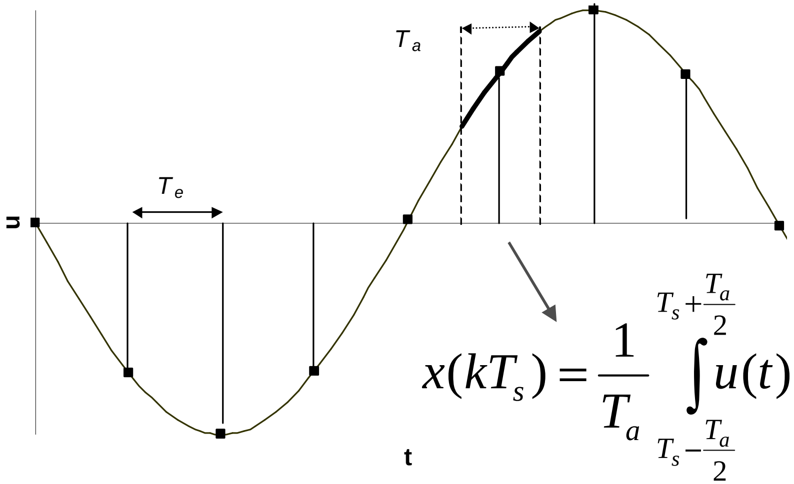


Digital sampling devices (6/18)

Configuration of voltmeters

Direct current reading sampling mode (DCV):

- aperture time T_a programmable,
- 50000 samples per second for T_a > 3 µs,
- BW ≈ 150 kHz for 1 V range,
- format DINT (4 bites) ⇒ + 5000 points.



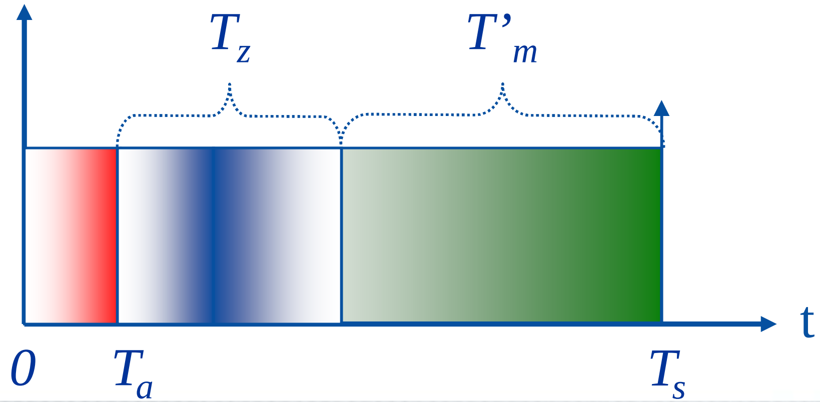
$$x(kT_s) = \frac{1}{T_a} \int_{T_s - \frac{T_a}{2}}^{T_s + \frac{T_a}{2}} u(t)dt$$

Ta (µs)	0.5	1	3	6	100
q	15	16	17	18	21

= an averaging of the input signal is made by the multimeter during T_a.

Parameters of influence:

- sampling period T_s (or frequency f_s),
- aperture time T_a ,
- dead time $T_d = T_s - T_a$,
- autozero and time needed for autozero T_z…..
- dead time with autozero $T'_m = T_e - T_a - T_Z$



Digital sampling devices (7/18)

Calculation of active power

$$\hat{x}(t,N) = \sum_{k=-\infty}^{+\infty} x(kT_e).\delta(t-kT_e) \xrightarrow[\text{Infinitely short pulses}]{\text{FT}} X(\nu) * F_e . \sum_{n=-\infty}^{+\infty} \delta(\nu - nF_e)$$

$\downarrow$ Pulses of width T_a

$$x(kT_e) = \frac{1}{T_a} x(t) * \pi_{\frac{T_a}{2}}(t) \Big|_{t=kT_e} \xrightarrow{\text{FT}} \underbrace{\left[X(\nu) \frac{\sin(\pi\nu T_a)}{\pi\nu T_a} \exp\left(-2\pi j \nu \frac{T_a}{2}\right)\right]}_{Y(\nu)} * F_e . \sum_{n=-\infty}^{+\infty} \delta(\nu - nF_e)$$

$\rightarrow$ Amplitude spectrum $\quad |Y(\nu)| = \sqrt{(\mathrm{Re}\{Y(\nu)\})^2 + (\mathrm{Im}\{Y(\nu)\})^2}$

$\rightarrow$ Phase spectrum $\quad \phi(\nu) = \arctan\left(\frac{\mathrm{Im}\{Y(\nu)\}}{\mathrm{Re}\{Y(\nu)\}}\right)$ **+ 0,5π** if $\mathrm{Re}\{Y(\nu)\} \geq 0$

< 0

Digital sampling devices (8/18)

Calculation of active power

Given: $A(0) = \mathrm{Re}\{Y(0)\}$ and $A(k) = 2|Y(\nu)|$ For k = 1, 2, …N-1.

$$\Rightarrow \quad x(t) = A(0) + \sum_{k=1}^{N-1} A(k)\cos\left[2\pi k \frac{t}{T} + \phi(k)\right]$$

if

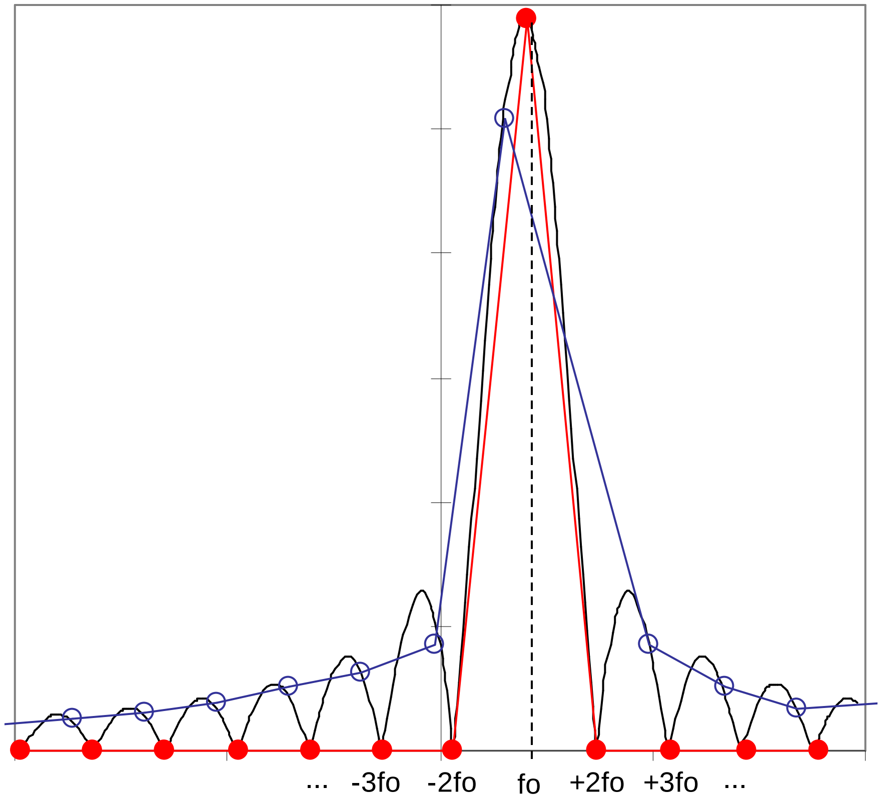
- x(t) periodical,
- Shannon theorem satisfied,
- *fs* = (N/M).*f* when N samples are taken over M periods.

For an exact measure of the active power : preceeding conditions + voltage and current samples must be taken **simultaneously**, at **equal time intervals**.

$\Rightarrow$ **Synchronization system**

Digital sampling devices (9/18)

Synchronization system



Finite support ⇒ windowing:
- time domain: x(t)×∏(t)
- frequency domain: X(f)*TF[∏(t)]

If *fs* = (N/M).*f* ⇒ discrete Dirac,
= no errors

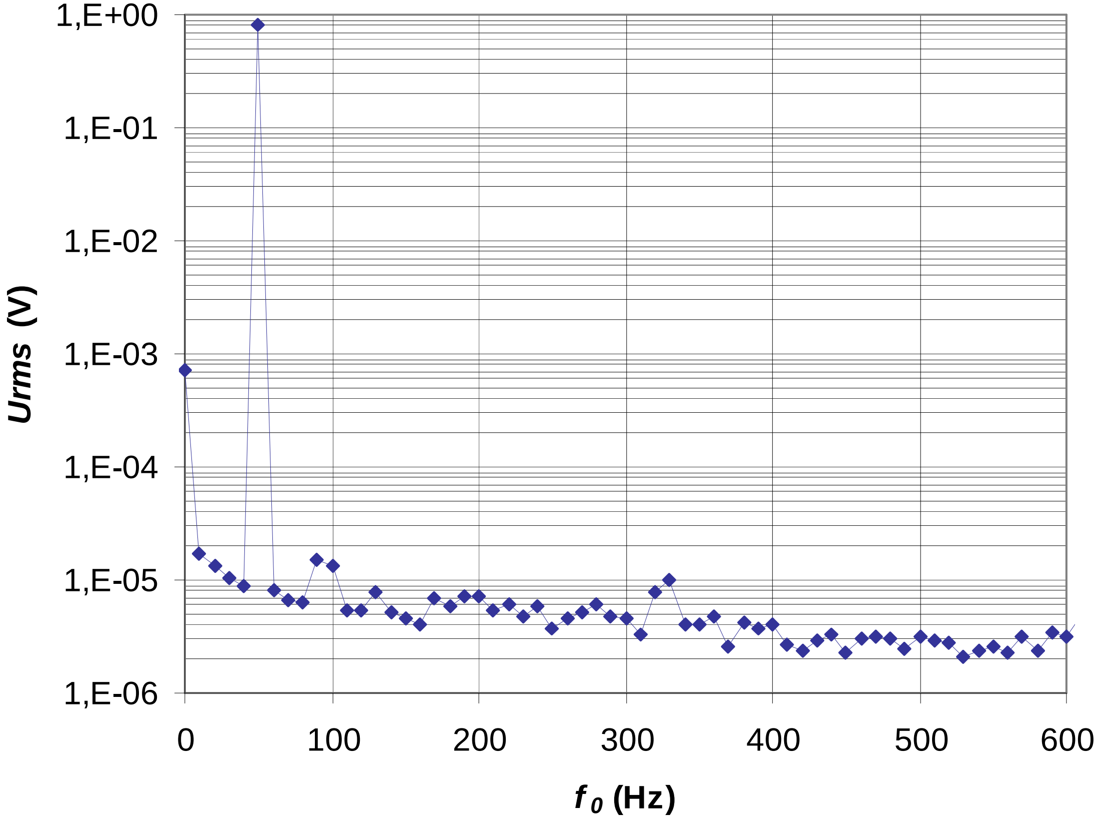
If *fs* ≠ (N/M).*f* ⇒ amplitude highly damped + broadening of the base of the pic
= **truncature** error : « **spectral leakage** ».

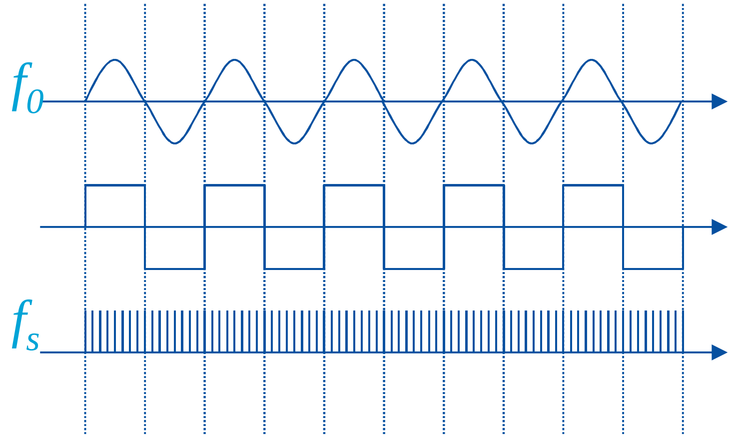
⇒ Use a **phase locked loop** (PLL).

Digital sampling devices (10/18)

Synchronization system

Signal amplitude spectrum





Quantization error

(S/N)dB = 6×q
q:coding number of bits = 18

⇓

For S = 1V,
10^{-6} V < N < 10^{-5} V.

⇒ **no truncature error** on the measurement of *u(t)* and *i(t)* at 53 Hz

Digital sampling devices (11/18)

Analysis of errors

$$P = UI\cos\phi \longrightarrow \frac{\Delta P}{P} = \frac{\Delta U}{U} + \frac{\Delta I}{I} - \tan(\phi)\Delta\phi$$

Phase errors

Quadrature errors

- **Mode DCV – $\mathrm{DMM_U}$ + $\mathrm{DMM_I}$ (1)**
- Band width limitation – $\mathrm{DMM_U}$ + $\mathrm{DMM_I}$
- **Aperture time T_a – $\mathrm{DMM_U}$ + $\mathrm{DMM_I}$ (2)**
- Sampling jitter
- Signal quantization
- Inductive divider ratio
- DC resistance
- AC-DC transfer difference

- Phase shift introduced by the inductive divider
- Phase shift introduced by the shunt
- **Phase shift introduced by the two digital wattmeters : band width difference + Δt difference between the trigger event and the 1rst sampling event + “sampling jitter” influence + influence of T_a (3)**
- Quantization noise

Primary standards

Digital sampling devices (12/18)

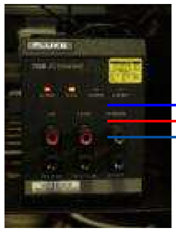
Analysis of errors

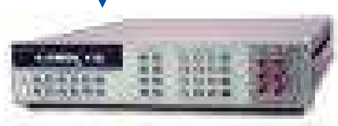
(1) – voltmeters calibration in DC mode

Functions synthesizer NF1930

(TTL signal)

Ext. Trig.





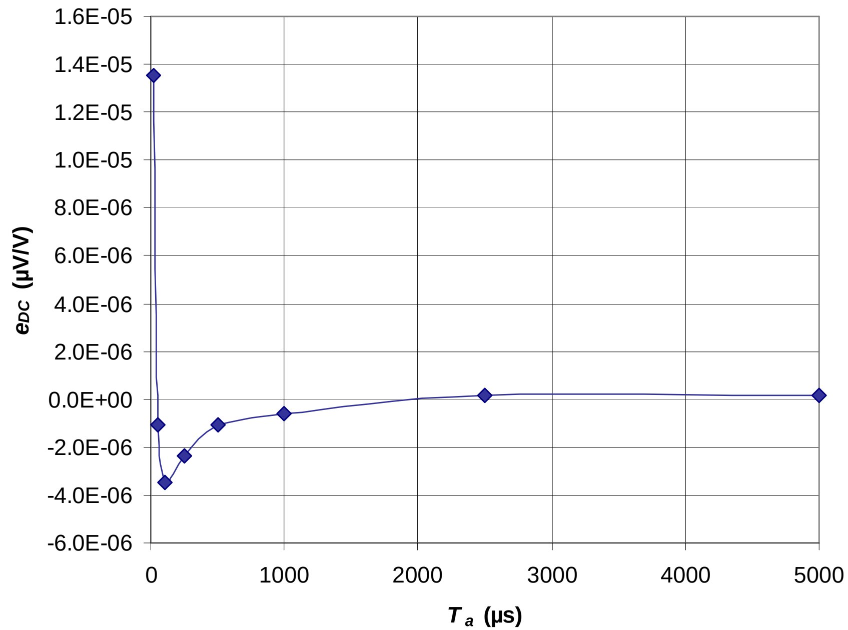
GPIB

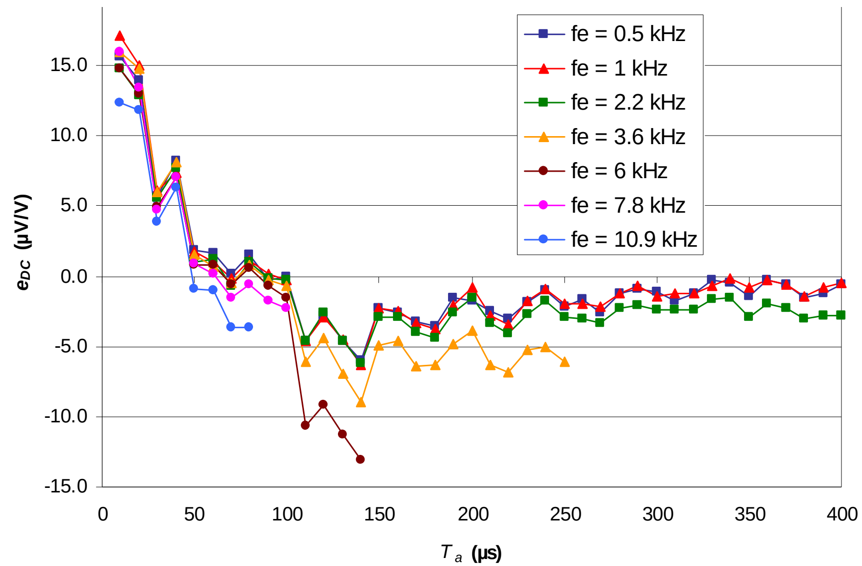
Fluke 732B

U_{ref} = 1,018167481 V

σ = 3,8.10^{-8} V

LabVIEW acquisition



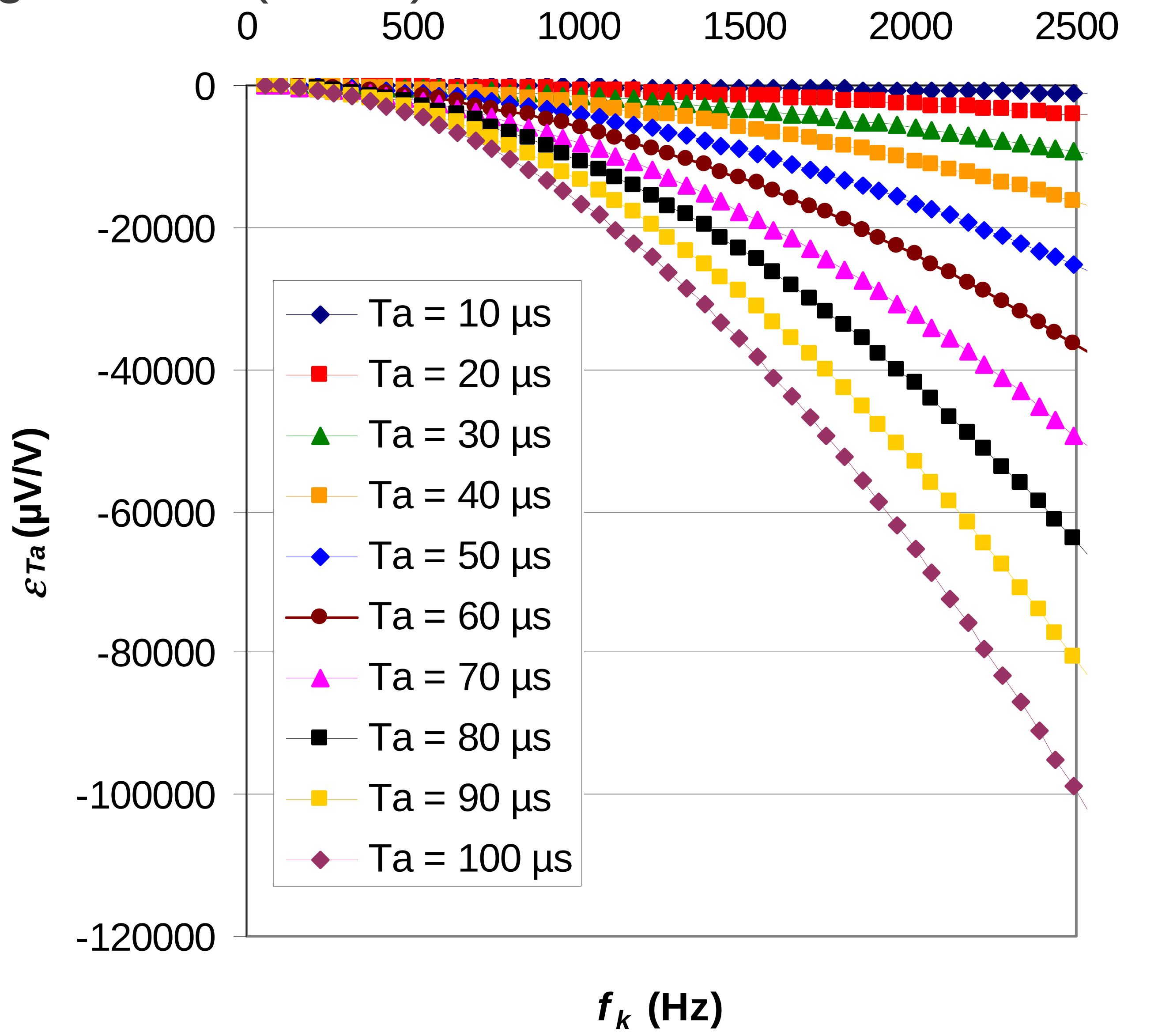


Digital sampling devices (13/18)

Analysis of errors

(2) Taking into account the aperture time of ADCs

$$X(\nu)\frac{\sin(\pi\nu T_a)}{\pi\nu T_a}$$



Digital sampling devices (14/18)

Analysis of errors

(3) – Quadrature errors due to the use of two multimeters

- Bandwidth difference:

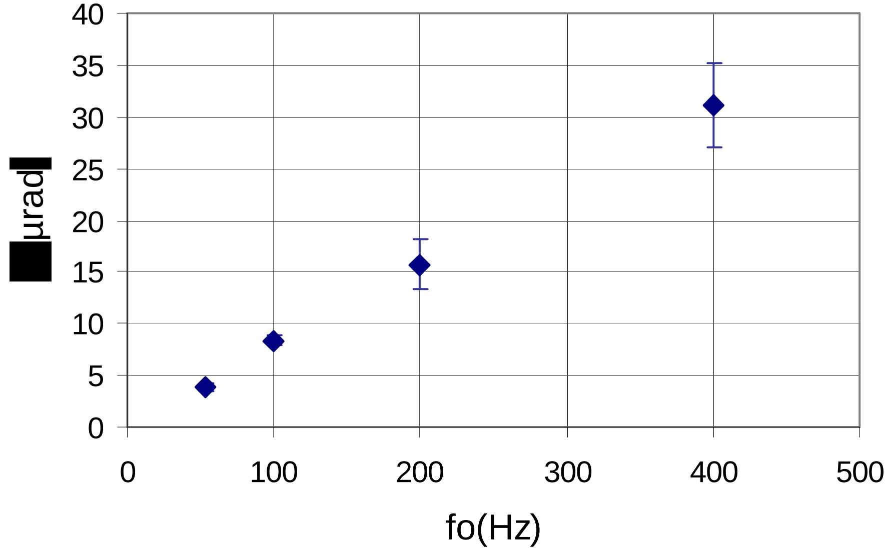
$$(\Delta\phi)_{BW} = \arctan\left(\frac{f}{f_1}\right) - \arctan\left(\frac{f}{f_2}\right) \approx \frac{f}{f_1} - \frac{f}{f_2}$$

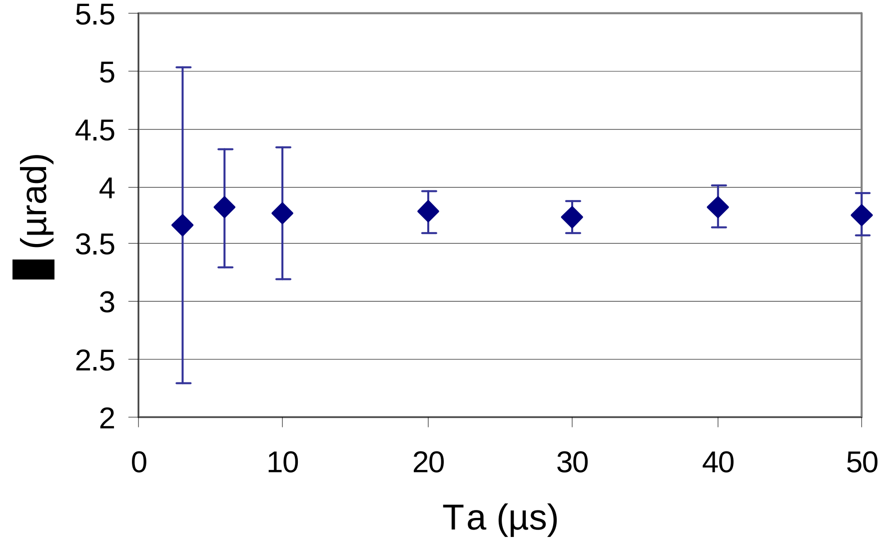
-Δt difference between the trigger event and the first sampling event:

$$(\Delta\phi)_{delay} = 2\pi f \Delta t$$

- Influence of the "sampling jitter" *(negligeable)*
- Influence of aperture time T_a

$$(\Delta\phi)_{Ta} = \pi[(\nu T_a)_U - (\nu T_a)_I]$$



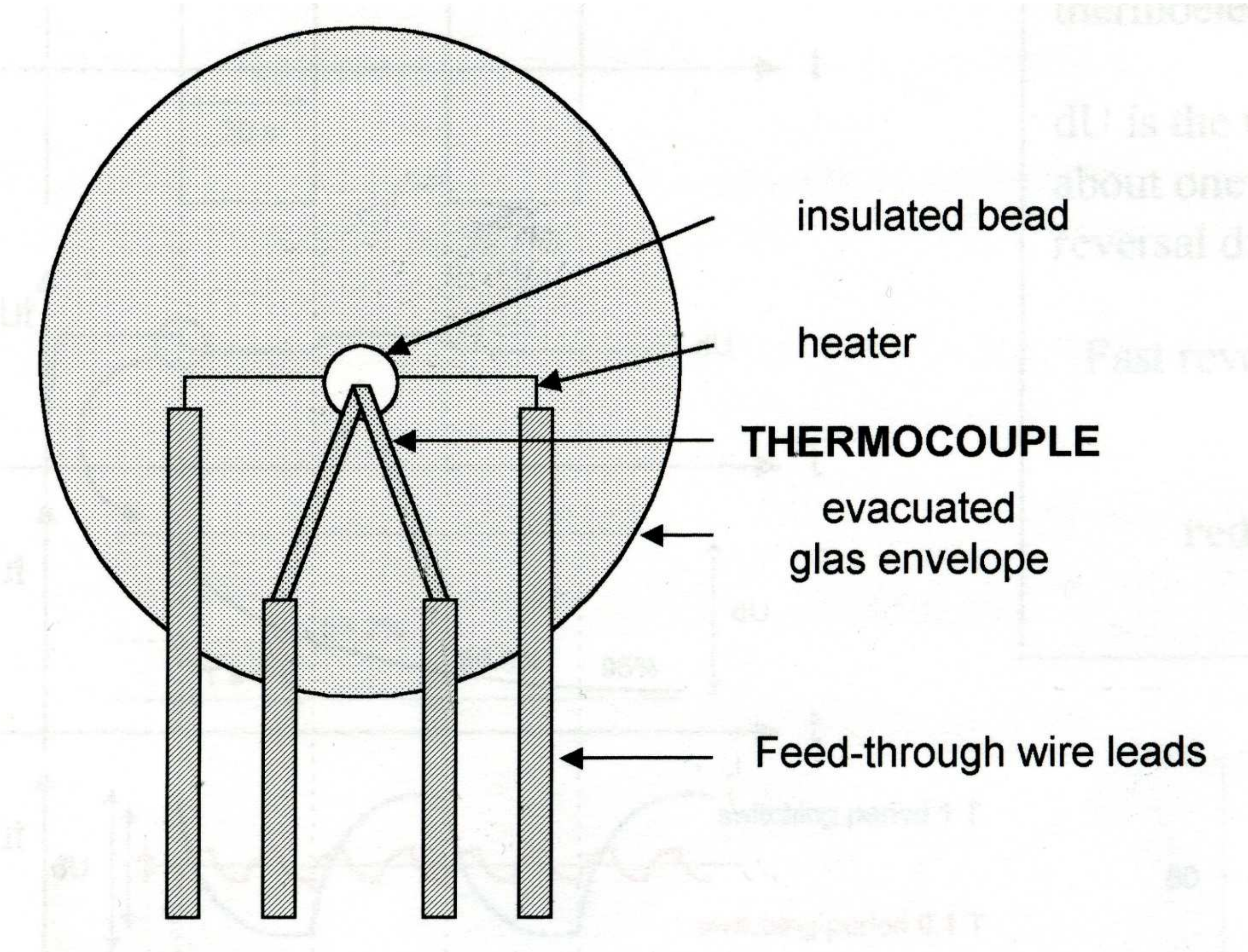


$\phi \pm \Delta\phi$ = (3,849 ± 0,031) µrad to f_o = 53 Hz et T_a = 20 µs

Digital sampling devices (15/18)

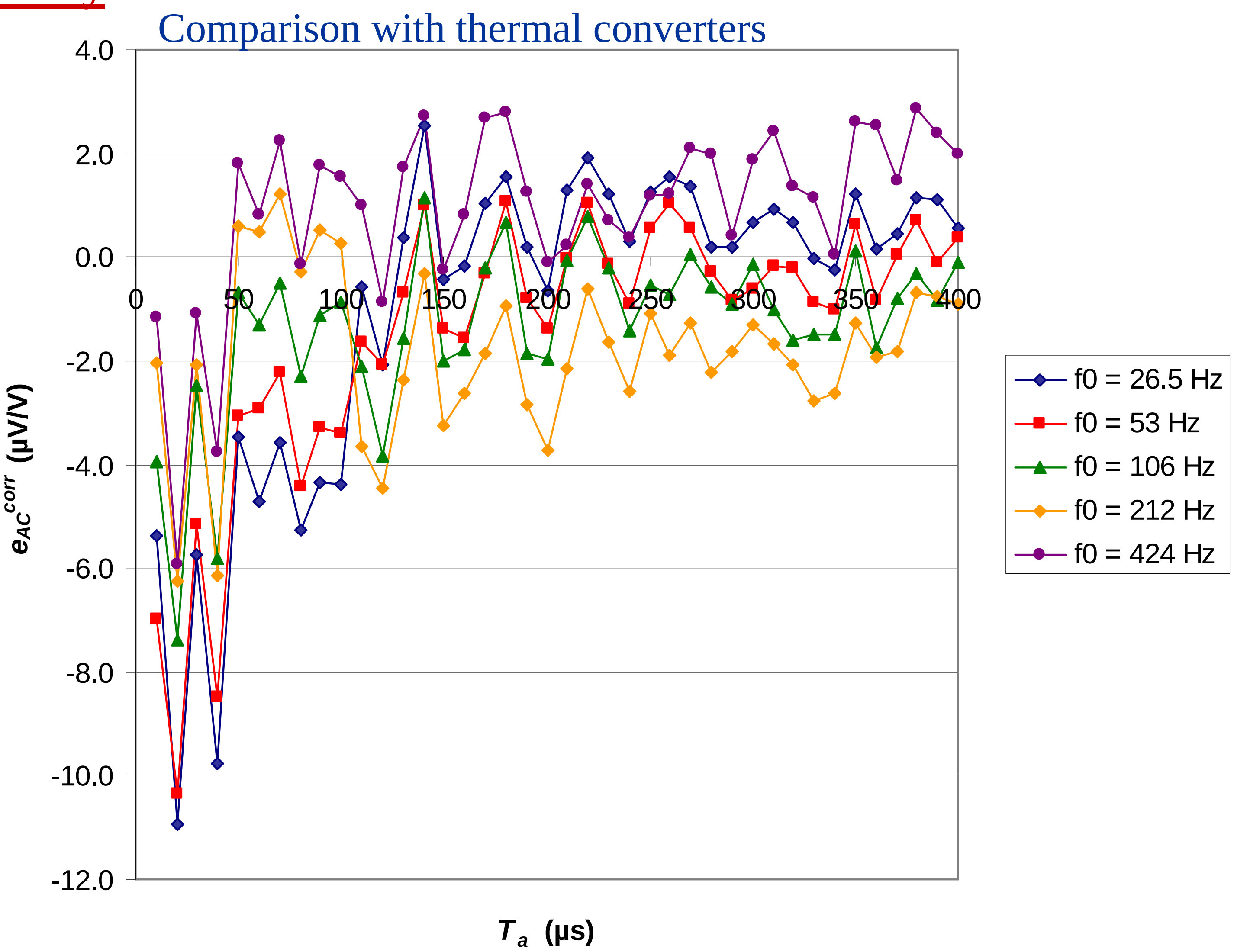
Traceability

Comparison with thermal converters
(it is usual to link alternative quantities to corresponding continuous ones using thermal converters)



Digital sampling devices (16/18)

Traceability



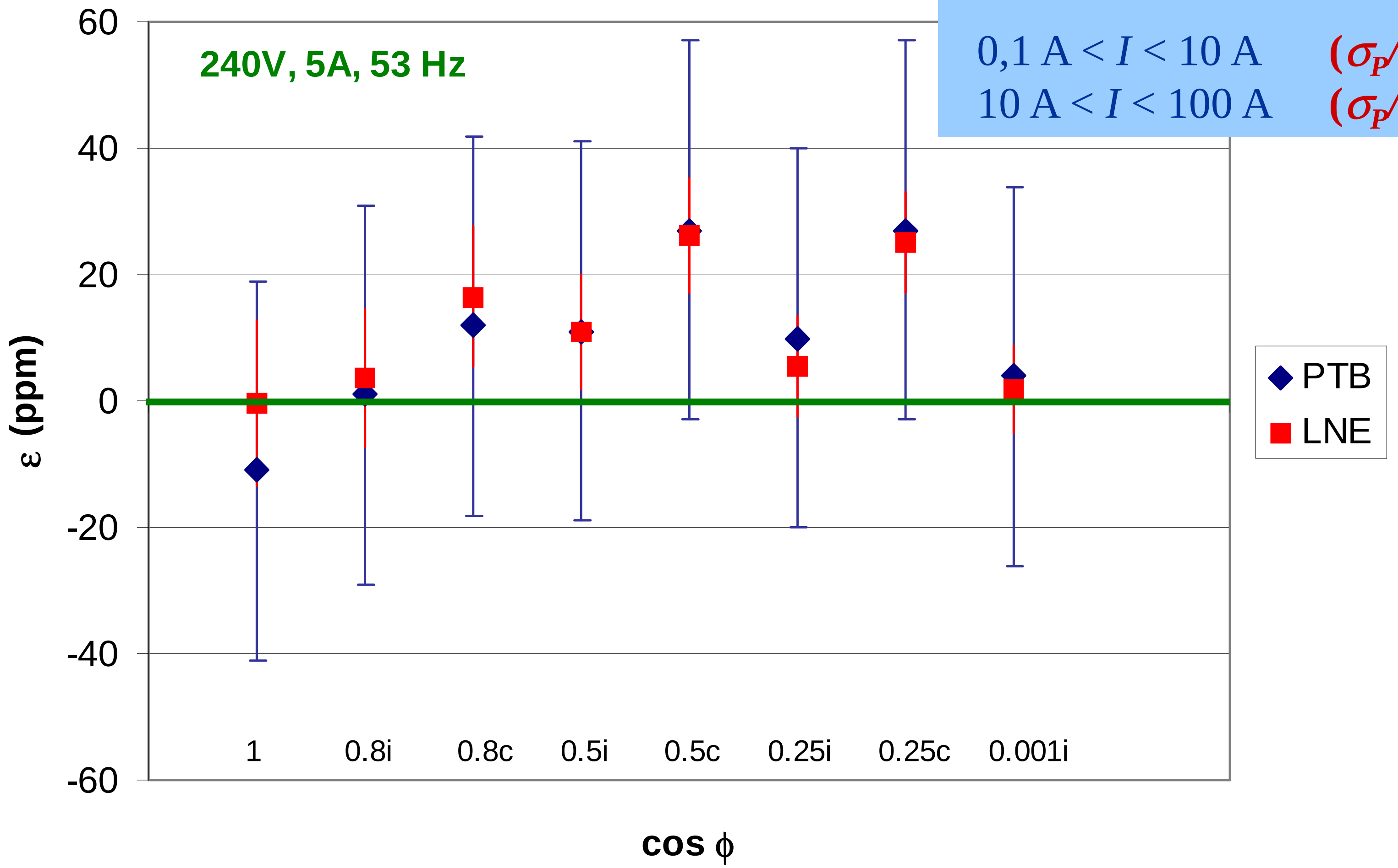
Digital sampling devices (17/18)

Bilateral comparison

(commercial wattmeter K2005 calibrated at PTB)

f_0 = 53 Hz, 60 V<U<600 V, cosϕ variable,

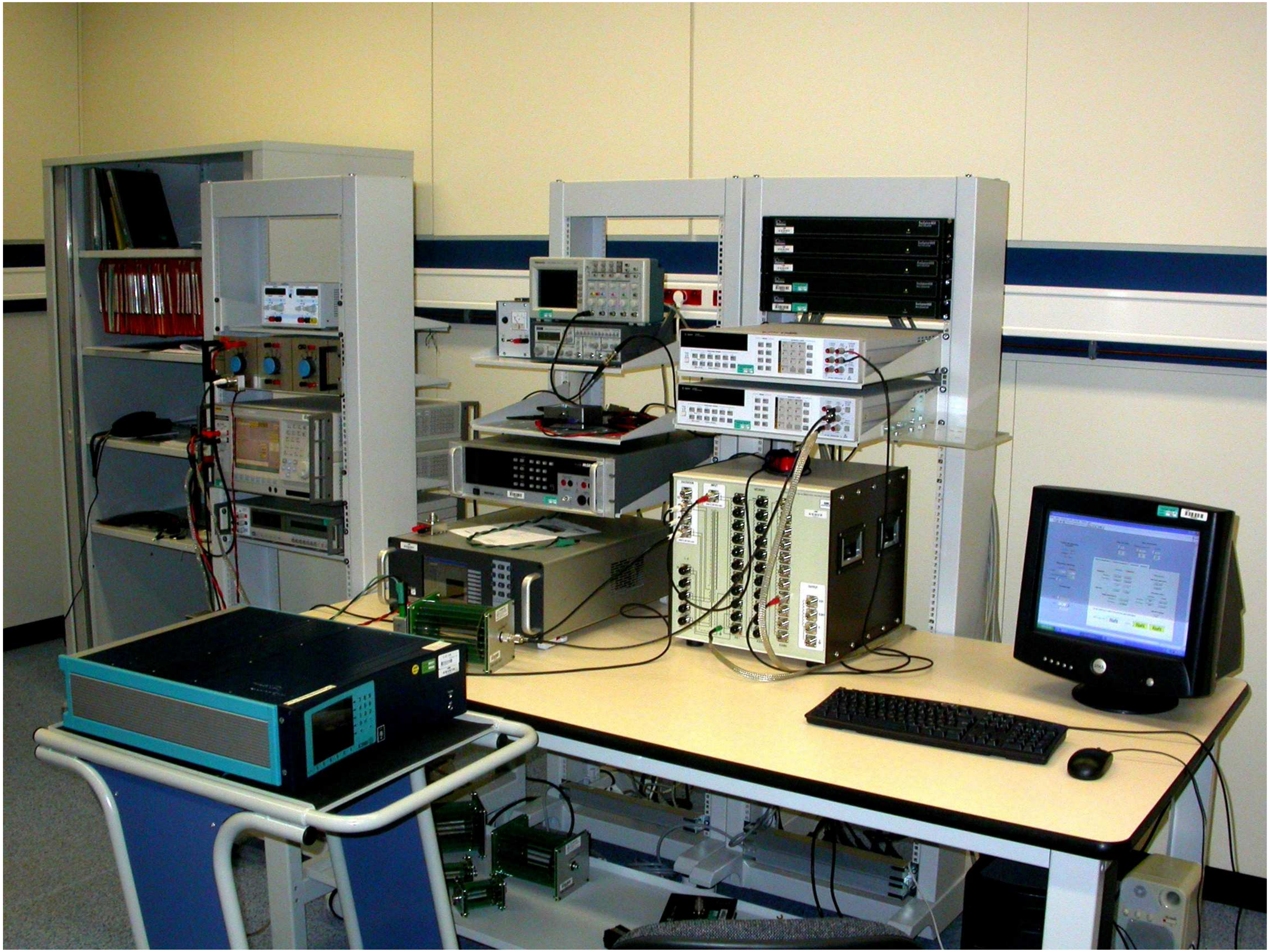
0,1 A < I < 10 A	**(σ_P/S) < 15 μW/VA**
10 A < I < 100 A	**(σ_P/S) < 30 μW/VA**

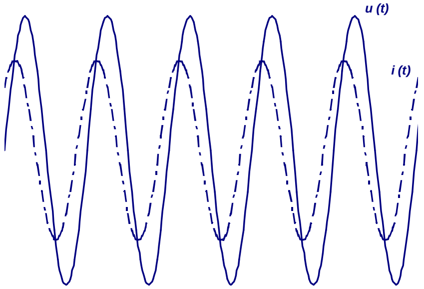


Digital sampling devices (18/18)

LNE standard uncertainty (k=1)
from 8 to 13 µW/VA
f =53 Hz, U = 120 V, I = 5 A
cos ϕ : from 0, 001 (inductive or capacitive) to 1

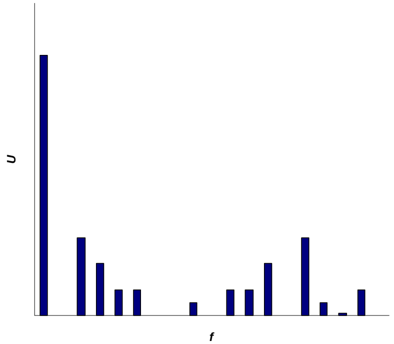
Precaution : a FFT processing requires an integer ratio sampling period / signal period



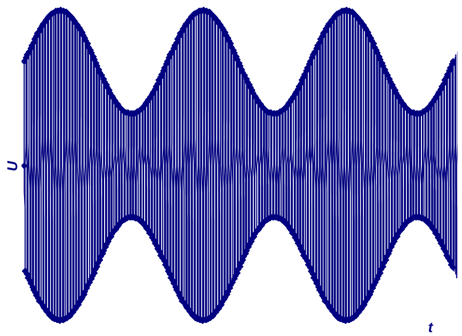


Measurement of U, I and power components S, P and Q, in AC sinusoid regime and at industrial frequencies (20 Hz $\leq f_0 \leq$ 400 Hz).

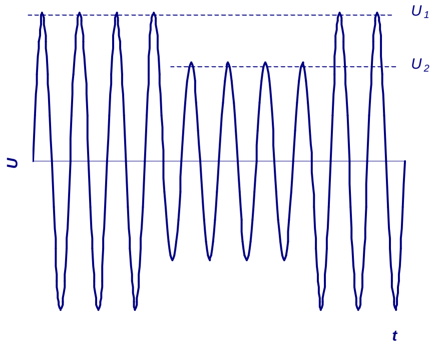
Measurement of U, I and harmonic distortion ratio *THD* in distorted regime in presence of quasi-stationary harmonics.



Measurements of Flicker in presence of amplitude square or sinusoid modulation.

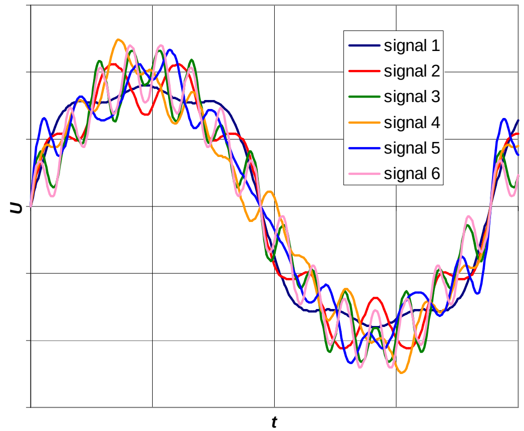


modulation d'amplitude sinusoïdale



modulation d'amplitude carrée

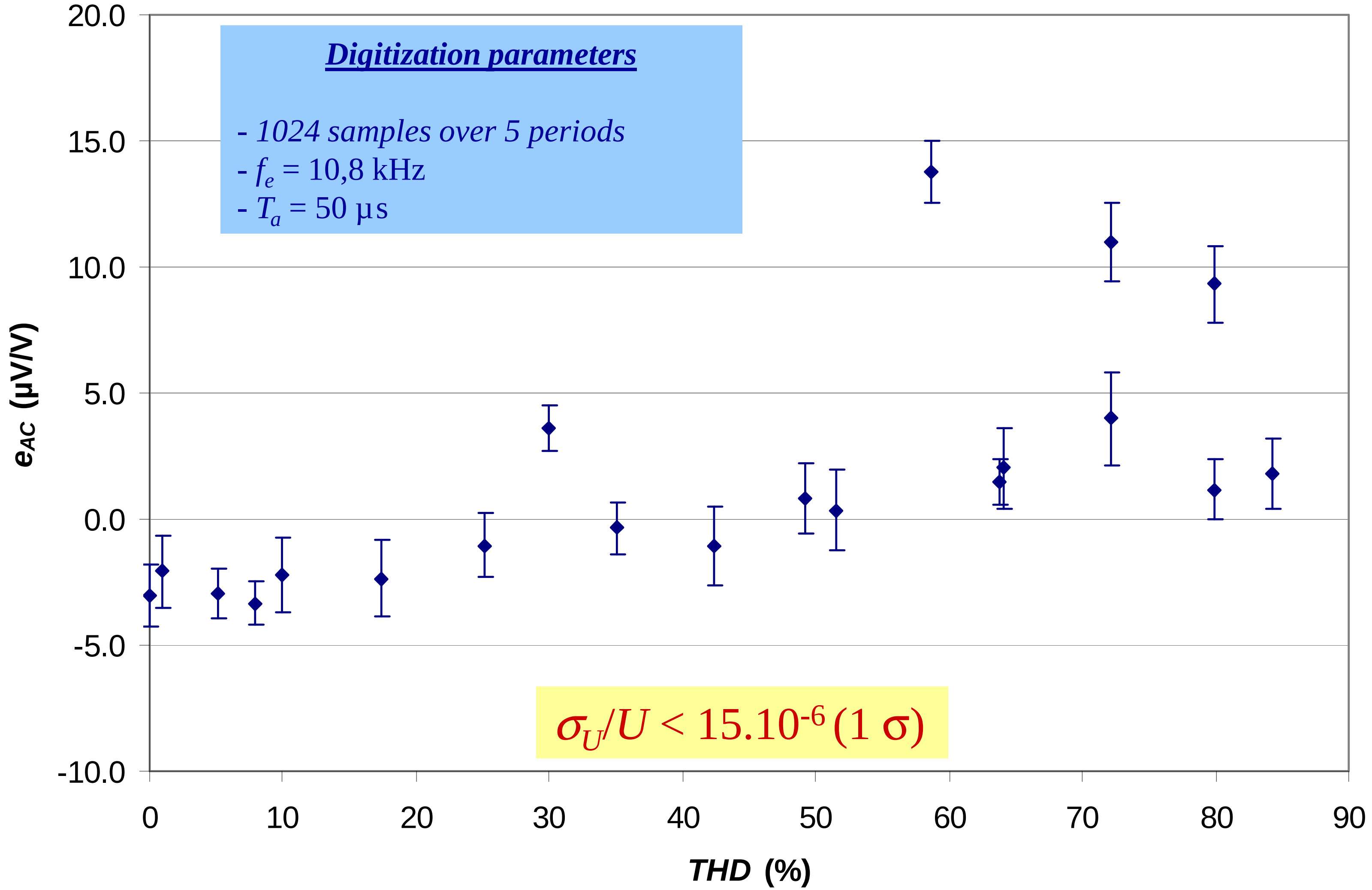
…and power in distorted regime



Power quality

Measurements of voltage in distorted regime (quasi-stationary harmonics

Studied signals: f_0 = 53 Hz, k_{max} = 40, 60 V < U < 600 V, N harmonics, THD variable.

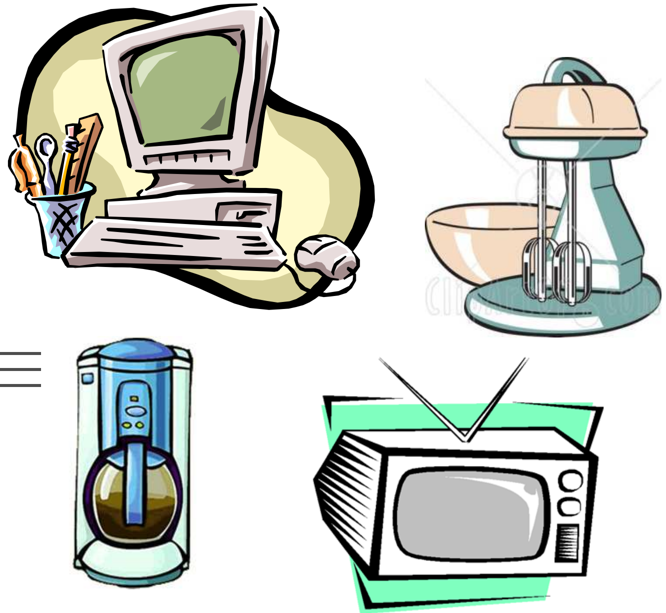


Flicker measurements

Flicker = slow variation of voltage due to variable loads.

Industrial equipments disturbances :
ex., variation of light intensity of illumination

Grid



Variable electric loads: arc furnaices, laser printers, microwaves….

⇒

⇒ negative effects on human beings (visual and nervous fatigue, headhache, irritability, epilepsy…).

Under laboratory conditions , **Flicker** = modulated signal resulting from either square or sinusoid amplitude modulation of a sinusoid enveloppe (IEC-61000-4-15 standard).

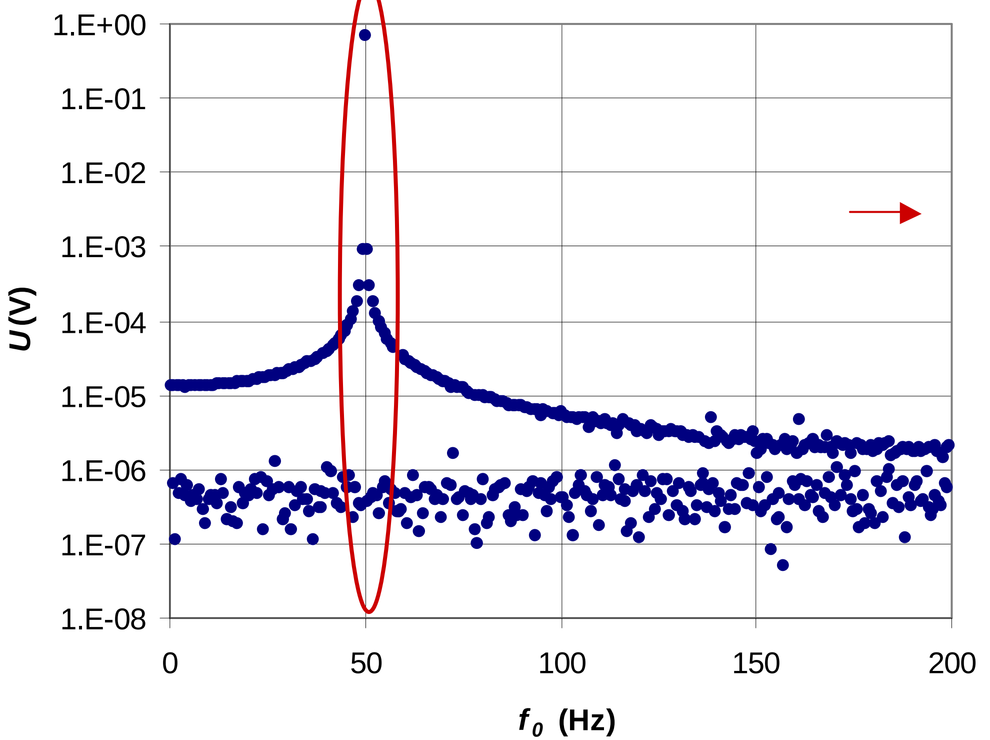
Flicker measurements

Square amplitude modulation

$$u^m(t) = U\sin(2\pi f_0 t) + \frac{2}{\pi}kU\sum_{\alpha=0}^{\infty}\frac{1}{(2\alpha+1)}\cos[2\pi(f_0-(2\alpha+1)f_m)t]$$
$$-\frac{2}{\pi}kU\sum_{\alpha=0}^{\infty}\frac{1}{(2\alpha+1)}\cos[2\pi(f_0+(2\alpha+1)f_m)t]$$

$\Rightarrow$ Measurement of **modulation factor k**

Spectral analysis $\Rightarrow$ an infinity of modulation lines at frequencies f_α with an amplitude of u_α : $f_\alpha = f_c \pm (2\alpha+1) f_m$



$$u_\alpha = \frac{2}{\pi}\frac{kU}{(2\alpha+1)}$$

$$f_0 \Rightarrow U \; et \; f_0 \pm f_m \; (\alpha = 0) \Rightarrow u_0$$

$$k = \frac{\pi}{2}\frac{u_0}{U}$$

Beware to aliasing

$\sigma_k / k = 1{,}1.10^{-5}$ (1 σ)

(for square and sinusoid modulations)

Power quality

Quantities measured at LNE: synthesis of results

- Measurements of *U, I, S, P, Q, cos* ϕ *and (sin* ϕ*)* in sinusoid regime and at industrial frequencies (20 Hz ≤ f_0 ≤ 400 Hz, *0,05 (***1 mA***) A < I < 100 A, U < 1000 V)*

- Measurements of *U, I* in distorted regime in presence of quasi-stationary harmonics and measurement of *THD* (f_0 = 53 Hz, k_{max} = 40)

- Measurements of Flicker *(conditions defined by IEC-61000-4-15 standard)*

Current developments

- Measurements of *U, I, S, P, Q* up to 10 kHz in sinusoid regime (ADCs of HP 3458)
Then up to 100 kHz in sinusoid regime (characterization of new ADCs: NI PXI-4462, NI PXI 5922)

- Power measurements in distorted regime (f_0 = 53 Hz and k_{max} = 40)

- Study of signals containing fluctuating harmonics (use of other calculation methods: short term Fourier transform, wavelets transform, Kalman filtering…)